LINKÖPINGS UNIVERSITET

Exam in TSKS14 Multiple Antenna Communications

Exam code: TEN1

Date: 2020-08-18 **Time:** 08:00–12:00

Place: Distance exam with special instructions:
Submit your solution as a single PDF document using the submission system in Lisam. The submission is open until 12:30, so you have 30 min to create a PDF. (If the submission system doesn't work, please send the exam solution to the teacher by email before 12:30.)
Write your name and personal number at every page and number the pages. Verify that your PDF is complete since no solutions are accepted after the deadline.

Teacher: Emil Björnson, emil.bjornson@liu.se

Visiting exam: The examiner is available to answer questions on 013 - 28 67 32.

Allowed aids: Due to the distance mode, all aids are allowed but all forms of cooperation with other people during the exam hours is strictly forbidden.
We recommend the following aids: Pocket calculator.
Björnson: Introduction to Multiple Antenna Communications.
Marzetta et al.: Fundamentals of Massive MIMO.
Olofsson: Tables and Formulas for Signal Theory.

Number of tasks: 5

Solutions: Will be published after the exam on the course web page.

Result: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

Exam return: The corrected exams will be sent back to the students in Lisam.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get the indicated number of points from each problem. At most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

The answer to each question must be supported by an argument or derivation; it is *not* enough to just answer yes/no or to give a number. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 The signal-to-noise ratio (SNR) determines how much information can be transmitted per modulation symbol in a wireless communication system. If the SNR is below a certain value, the system is not operational and we say that we are out-of-coverage. In this question, we consider a system where the SNR must be above $-10\,\text{dB}$ for the system to be operational. (5p)

a. Consider a system with a single-antenna base station that communicates with a single-antenna user terminal. The base station transmits with $10\,\text{W}$ and the terminal transmits with $0.1\,\text{W}$. The channel gain is $-110\,\text{dB}$, the bandwidth is $10\,\text{MHz}$, and the noise power spectral density is $10^{-17}\,\text{W/Hz}$.

Compute the SNRs achieved in the uplink and the downlink. (1p)

b. The computation in Part **a** shows that the uplink SNR is below $-10\,\text{dB}$. Hence, the system is not operational, even if the downlink SNR is above $-10\,\text{dB}$. This is a common issue in practical systems.

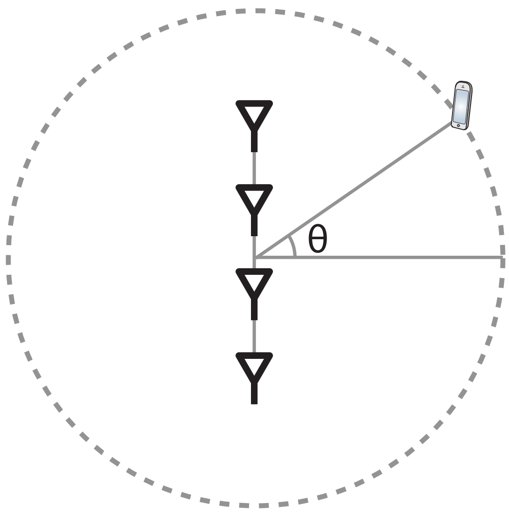
Your task is to identify one or multiple ways to address this issue so that the resulting uplink SNR is above the threshold.

(a) Can we use multiple antennas at the base station? If yes, how many antennas are needed in this case? If no, explain why. (2p)

(b) Can we change how much bandwidth that is used? If yes, explain how and what the consequences will be. If no, explain why. (2p)

2 Write a text where you explain, with your own words, what the beamwidth is and when it is important to have a small beamwidth in a wireless communication system. Describe how one can achieve a small beamwidth. Give a mathematical example of how to compute the beamwidth with for an array of M antennas (you are free to choose the geometry of the array). How many antennas are needed to get a beamwidth of $\approx 15^\circ$ in your example? (5p)

3 A base station equipped with $M = 4$ antennas is transmitting to a single-antenna device that is located at an angle θ, as illustrated in this figure: (5p)



The channel vector $\boldsymbol{g}$ is deterministic and depends on θ. It is modeled as

$$\boldsymbol{g} = \begin{bmatrix} 1 & e^{j\pi \sin(\theta)} & e^{j2\pi \sin(\theta)} & e^{j3\pi \sin(\theta)} \end{bmatrix}^T.$$

a. Suppose the capacity of this channel is $\log_2(41)$ when the channel is perfectly known. What is the SNR ρ? (1p)

b. Give an expression of a precoding vector that can be used to achieve the capacity. Explain how the precoding vector and the capacity value depends on θ. (1p)

c. Suppose the base station believes that the user is located at $\theta = 0^\circ$ and transmits with the precoding that would have achieved the capacity in that case. If the true angle of the user is $\theta_1 = 60^\circ$, what is the achievable information rate? Compare the result with Part **a**. (2p)

d. Repeat Part **c** but with $\theta_1 = 30^\circ$. Explain the result. (1p)

4 In this problem, we analyze how many terminals can be served in a multi-cell Massive MIMO and how high uplink performance can be achieved with maximum ratio processing. We consider a two-cell system where the same K pilots are used in both cells. We set $\eta_{lk} = 1$ for $l = 1, 2$ and $k = 1, \dots, K$. Moreover, all the intra-cell channels have large-scale fading coefficients $\beta_{1k}^{1} = \beta_{2k}^{2} = 1$ for $k = 1, \dots, K$, while the inter-cell channels have the large-scale fading coefficients $\beta_{1k}^{2} = \beta_{2k}^{1} = \alpha$ for $k = 1, \dots, K$. (5p)

a. Explain why the uplink effective SINR is the same for all terminals and provide the corresponding expression. What is the resulting lower bound on the capacity?

(Hint: The expression will depend on M, K, and ρ_{ul}.) (2p)

b. Suppose each coherence interval has length τ_{c}, then only a fraction $(1 - K/\tau_c)$ of it is used for uplink data transmission. What is the net sum spectral efficiency in each cell?

(Hint: Spectral efficiency is the same thing as a lower bound on the capacity) (1p)

c. What happens to the net sum spectral efficiency in Part **b** as $M \to \infty$? (1p)

d. Consider the asymptotic limit from Part **c**. For which value of K is the expression maximized? (1p)

5 Consider the channel (5p)

$$y[l] = g \cdot x[l] + w[l],$$

where l is the time index, $\{x[l]\}$ is the input sequence with power P, $\{y[l]\}$ is the output sequence, and $\{w[l]\}$ is i.i.d. $CN(0, 1)$ noise. The channel coefficient g is uniformly distributed between -1 and $+1$. We consider a slow-fading scenario where the value of g is only known at the receiver. Answer the following questions:

a. Derive the outage probability of this channel. Express the answer as a function of the desired rate $R \geq 0$. (2p)

b. Suppose we have two receive antennas and these observe *the same* channel realization g. What is the outage probability for the desired rate $R \geq 0$? (1p)

c. Derive expressions for the ϵ-outage capacities for the setups considered in Parts **a** and **b**. Sketch a graph of the two expressions with ϵ on the horizontal axis and the ϵ-outage capacity on the vertical axis. (2p)

Table of formulas

Capacity lower bound for the kth terminal in a single-cell system:

Uplink with arbitrary decoding vector:

$$E\left\{\log_2\left(1+\frac{\rho_{\mathrm{ul}}\eta_k|\mathbf{a}_k^H\hat{\mathbf{g}}_k|^2}{\mathbf{a}_k^H\left(\sum_{\substack{k'=1\\k'\neq k}}^{K}\rho_{\mathrm{ul}}\eta_{k'}\hat{\mathbf{g}}_{k'}\hat{\mathbf{g}}_{k'}^H+\left(\sum_{k'=1}^{K}\rho_{\mathrm{ul}}\eta_{k'}(\beta_{k'}-\gamma_{k'})+1\right)\mathbf{I}_M\right)\mathbf{a}_k}\right)\right\}$$

Downlink with arbitrary precoding vectors:

$$\log_2\left(1+\frac{\rho_{\mathrm{dl}}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}{\sum_{k'=1}^{K}\rho_{\mathrm{dl}}\eta_{k'}E\{|\mathbf{g}_k^T\mathbf{a}_{k'}|^2\}+1-\rho_{\mathrm{dl}}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}\right)$$

Effective SINR for the kth terminal in the lth cell in a multi-cell system:

Uplink maximum-ratio:

$$\mathrm{SINR}_{lk}^{\mathrm{mr,ul}}=\frac{M\rho_{\mathrm{ul}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+\rho_{\mathrm{ul}}\sum_{l'\notin\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+M\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{l'k}^{l}\eta_{l'k}}$$

Downlink maximum-ratio:

$$\mathrm{SINR}_{lk}^{\mathrm{mr,dl}}=\frac{M\rho_{\mathrm{dl}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{dl}}\sum_{l'\in\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^{K}\eta_{l'k'}\right)+\rho_{\mathrm{dl}}\sum_{l'\notin\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^{K}\eta_{l'k'}\right)+M\rho_{\mathrm{dl}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{lk}^{l'}\eta_{l'k}}$$