Exam in TSKS14 Multiple Antenna Communications

Exam code: TEN1

Date: 2020-06-02 **Time:** 08:00–12:00

Place: Distance exam with special instructions:
Submit your solution as a single PDF document using the submission system in Lisam. The submission is open until 12:30, so you have 30 min to create a PDF. (If the submission system doesn't work, please send the exam solution to the teacher by email before 12:30.)
Write your name and personal number at every page and number the pages. Verify that your PDF is complete since no solutions are accepted after the deadline.

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Visiting exam: The examiner is available in the chat on Microsoft Teams.

Allowed aids: Due to the distance mode, all aids are allowed but all forms of cooperation with other people during the exam hours is strictly forbidden.
We recommend the following aids: Pocket calculator.
Björnson: Introduction to Multiple Antenna Communications.
Marzetta et al.: Fundamentals of Massive MIMO.
Olofsson: Tables and Formulas for Signal Theory.

Number of tasks: 5

Solutions: Will be published after the exam on the course web page.

Result: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

Exam return: The corrected exams will be sent back to the students in Lisam.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get the indicated number of points from each problem. At most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

The answer to each question must be supported by an argument or derivation; it is *not* enough to just answer yes/no or to give a number. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Ericsson published a press release in February 2020 about a demonstration of their new 5G products that operate in the millimeter wave spectrum. In particular, they achieved a data rate of 4.3 Gbit/s over a point-to-point channel using a bandwidth of 800 MHz. The purpose of this problem is to analyze how this data rate might have been achieved. (5p)

a. Suppose a free-space line-of-sight SISO channel was used with perfect knowledge of the channel at the receiver. If the capacity was achieved, what must have been the SNR? (1p)

b. If isotropic antennas are used in the SISO channel, the received signal power is given by

$$P\frac{\lambda^2}{(4\pi)^2}\frac{1}{d^2}$$

where the wavelength λ is 10 mm and the transmit power is 100 W. How large was the propagation distance d if the noise power spectral density is $N_0 = 10^{-17}$ W/Hz? (1p)

c. Suppose the transmitter was equipped with 8 isotropic antennas and the receiver was equipped with 2 isotropic antennas. We still consider a free-space line-of-sight channel but now with uniform linear arrays. Explain how to compute the channel capacity in this case, including how the transmitter creates the transmitted signal and how the receiver processes the received signal. How is the SNR computed? How large was the propagation distance d? (3p)

2 Some 5G deployments are making use of spectrum in millimeter wave bands. The benefit of doing so is the huge bandwidths that are available, for example, 800 MHz as in the example given in Problem 1. This can be compared with the 10-40 MHz of spectrum that is common in the conventional frequency bands used for wireless communications. The drawback of millimeter wave bands is that the signals are easily blocked. If the user happens to hold his/her hand over the antennas, there might not be any signal at all. (5p)

In this problem, we consider the downlink transmission from a base station with M antennas to a single-antenna user terminal. The channel vector is modeled as

$$\boldsymbol{g} = b\boldsymbol{g}_{\text{fixed}}$$

where $\boldsymbol{g}_{\text{fixed}}$ is a deterministic and known vector while b is a random number that is 1 with probability p and 0 with probability $1-p$. This variable models the probability that the user happens to block the signal.

a. Consider a fast fading channel where a new independent realization of b is drawn in each coherence interval. Compute a lower bound on the capacity using the formula "Downlink with arbitrary precoding vectors" on the last page of the exam. The precoding vector $\boldsymbol{a}$ has unit norm and is independent of b. Which choice of precoding vector maximizes the capacity bound? (2p)

b. Compare the result from **a** with the capacity of the deterministic channel obtained if $p = 1$. Propose a way to compensate for the performance loss when $p < 1$. (1p)

c. Consider a slowly fading channel where a single realization of b appears during the transmission. Compute the outage probability, assuming that $\boldsymbol{g}$ and $\boldsymbol{a}$ are known at the receiver. Which choice of the unit-norm precoding vector $\boldsymbol{a}$ will minimize the outage probability for a given rate R? (2p)

3 Write a text that explains what a coherence interval is. The focus should be on how the carrier frequency f_c affects the size of the coherence interval. (3p)

Compare two network deployments: using a conventional frequency band with $f_c = 3$ GHz or using a millimeter wave band with $f_c = 30$ GHz. How will the size of the coherence intervals differ, if both networks support the same vehicular mobility and are deployed at the same place?

Which of the frequency bands is preferable and why?

4 Consider the downlink of single-cell Massive MIMO system. All users have the large-scale fading coefficient: $\beta_k = \beta$ for $k = 1, \dots, K$. (7p)

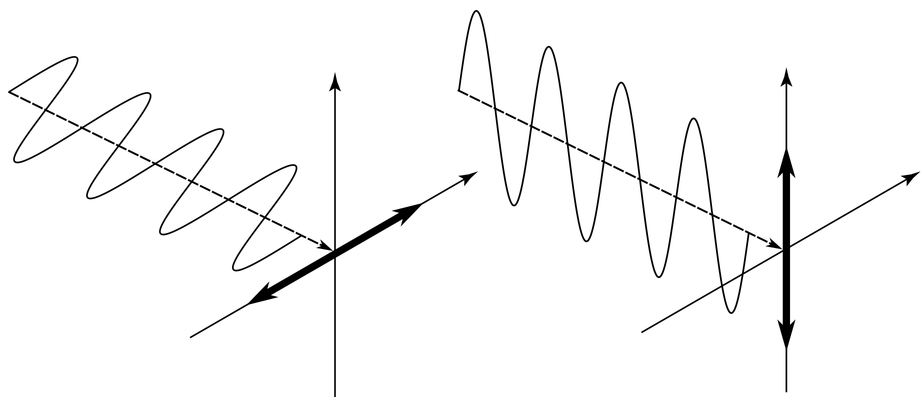
a. What is the net sum spectral efficiency of the system if $\tau_{\mathrm{p}} = K$ and the lower bound with maximum ratio processing is used? Simplify the expression so that it only consists ρ_{dl}, ρ_{ul}, β, M, K, τ_{c}, and the power control coefficients. (1p)

b. Derive the power control coefficients that maximizes the net sum spectral efficiency, under the usual power constraint $\sum_{k'=1}^{K} \eta_{k'} \leq 1$.
(Hint: Waterfilling can be utilized.)
What is the maximum net sum spectral efficiency? (2p)

c. Suppose the uplink power is low so that $1 + K\rho_{\mathrm{ul}}\beta \approx 1$ is an accurate approximation. Use the result from **b** and find the value of K that maximizes the net spectral efficiency (apply the approximation first). What is the maximum net sum spectral efficiency? (2p)

d. Use the result from **c** to compare a system with $\tau_{\mathrm{c}} = 400$ and a system with $\tau_{\mathrm{c}} = 100$. If all other parameters are the same, which system can achieve the highest net sum spectral efficiency? How many times larger is the net sum spectral efficiency? Explain the intuition behind this result. (2p)

5 Electromagnetic waves can have different polarization, describing in which direction the electrical field is varying. In the following figure, both waves propagate in the same direction, but the left example shows a horizontally polarized wave and the right example shows a vertically polarized wave: (5p)



A transmit antenna can only transmit signals of a predetermined polarization. A receive antenna can only observe the part of the received signal that has a polarization that matches with the antenna's own polarization.

a. Consider a line-of-sight point-to-point MIMO channel with two transmit antennas and two receive antennas. The odd-numbered antennas can transmit/receive horizontally polarized waves and the even-numbered antennas can transmit/receive vertically polarized waves. Consequently, the channel matrix can be written as

$$\boldsymbol{G} = \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix}.$$

Give an expression for the capacity of this channel by assuming the non-zero elements of $\boldsymbol{G}$ are computed as for line-of-sight channels (see Section 3 of the compendium). $\boldsymbol{G}$ is known everywhere. (2p)

b. Compare the result from **a** with the type of line-of-sight point-to-point MIMO channels with uniform linear arrays that were covered in the course material. Which of the channels is preferred and when (i.e., consider both low and high SNRs)? (1p)

c. Extend the analysis from **a** to a setup with M transmit antennas and K receive antennas, deployed as uniform linear arrays. M and K are even numbers. Note that $g_{m,k} = 0$ if m is odd and k is even, or if m is even and k is odd. (2p)

Table of formulas

Capacity lower bound for the kth terminal in a single-cell system:

Uplink with arbitrary decoding vector:

$$E\left\{\log_2\left(1+\frac{\rho_{\mathrm{ul}}\eta_k|\mathbf{a}_k^H\hat{\mathbf{g}}_k|^2}{\mathbf{a}_k^H\left(\sum_{\substack{k'=1\\k'\neq k}}^{K}\rho_{\mathrm{ul}}\eta_{k'}\hat{\mathbf{g}}_{k'}\hat{\mathbf{g}}_{k'}^H+\left(\sum_{k'=1}^{K}\rho_{\mathrm{ul}}\eta_{k'}(\beta_{k'}-\gamma_{k'})+1\right)\mathbf{I}_M\right)\mathbf{a}_k}\right)\right\}$$

Downlink with arbitrary precoding vectors:

$$\log_2\left(1+\frac{\rho_{\mathrm{dl}}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}{\sum_{k'=1}^{K}\rho_{\mathrm{dl}}\eta_{k'}E\{|\mathbf{g}_k^T\mathbf{a}_{k'}|^2\}+1-\rho_{\mathrm{dl}}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}\right)$$

Effective SINR for the kth terminal in the lth cell in a multi-cell system:

Uplink maximum-ratio:

$$\mathrm{SINR}_{lk}^{\mathrm{mr,ul}}=\frac{M\rho_{\mathrm{ul}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+\rho_{\mathrm{ul}}\sum_{l'\notin\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+M\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{l'k}^{l}\eta_{l'k}}$$

Downlink maximum-ratio:

$$\mathrm{SINR}_{lk}^{\mathrm{mr,dl}}=\frac{M\rho_{\mathrm{dl}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{dl}}\sum_{l'\in\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^{K}\eta_{l'k'}\right)+\rho_{\mathrm{dl}}\sum_{l'\notin\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^{K}\eta_{l'k'}\right)+M\rho_{\mathrm{dl}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{lk}^{l'}\eta_{l'k}}$$