Exam in TSKS14 Multiple Antenna Communications

Exam code:	TEN1
Date:	2019-05-29 **Time:** 08:30–12:30
Place:	Hammingrummet
Teacher:	Emil Björnson, emil.bjornson@liu.se
Visiting exam:	-
Administrator:	Carina Lindström, 013 - 28 44 23, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Pocket calculator with empty memory. Olofsson: Tables and Formulas for Signal Theory.
Number of tasks:	5
Solutions:	Will be published after the exam on the course web page.
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2019-05-12, 12:45–13:00, Emil Björnson's office, Building B, top floor, corridor A between entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

The answer to each question must be supported by an argument or derivation; it is *not* enough to just answer yes/no or to give a number. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Consider a wireless communication system where the transmit power is P [W] and the bandwidth is B [Hz]. The transmitter has one antenna and the receiver has M antennas. There is an AWGN channel with a known channel gain between the transmitter and each of the receive antennas. (5p)

Assume a line-of-sight channel with isotropic antennas where the received signal power is at each antenna is

$$P \frac{\lambda^2}{(4\pi)^2} \frac{1}{d^2},$$

where λ is the wavelength and d is the propagation distance.

a. Give an expression for the channel capacity, as a function of M, P, B, λ, d, and the noise power spectral density N_0. (Make reasonable assumptions if needed.) (1p)

b. Suppose $M = 1$, $P = 1\,\text{W}$, $B = 10\,\text{MHz}$, $\lambda = 10\,\text{cm}$, and $N_0 = 10^{-17}$ W/Hz. At what distance d is the channel capacity 10 Mbit/s? (1p)

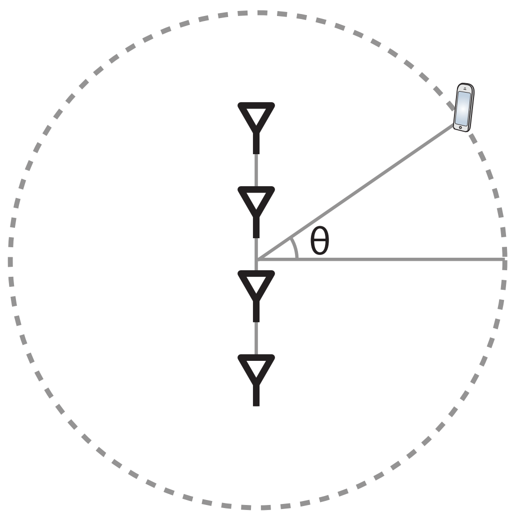
c. We want to increase the number of antennas to achieve a channel capacity of 100 Mbit/s at the same distance as in Part **b**. How many antennas are needed, if the other parameters are fixed? (1p)

d. The effective area of an isotropic antenna is $A_{\text{iso}} = \frac{\lambda^2}{4\pi}$. What is the total antenna area at the receiver in Part **c**? (1p)

e. We now reduce the wavelength to $\lambda = 1\,\text{cm}$. How many antennas are needed to achieve a capacity of 100 Mbit/s at the same distance as in Part **b**? How large is the total receive antenna area in this case? (1p)

2 Write a text that explains the capacity analysis for slow fading channels. Make sure to explain what slow fading is (as compared to fast fading) and to mathematically define both the outage probability and the outage capacity. Give an example where you compute the outage probability (for a fading channel of your choice) and then explain how multiple antenna technology can improve the performance. (5p)

3 A base station equipped with $M = 4$ antennas is transmitting to a single-antenna device that is located at an angle θ, as illustrated in this figure: (5p)



The channel vector $\boldsymbol{g}$ is deterministic and depends on θ. It is modeled as

$$\boldsymbol{g} = \begin{bmatrix} 1 & e^{j\pi \sin(\theta)} & e^{j2\pi \sin(\theta)} & e^{j3\pi \sin(\theta)} \end{bmatrix}^T.$$

a. What is the capacity of this channel, assuming that the SNR is $\rho = 2$ and that the channel is known perfectly? How does the capacity value depend on θ? (1p)

b. Suppose the base station believes that the user is located at $\theta = 0°$ and transmits using maximum ratio processing. If the true angle is $\theta = \theta_1$, what is the achievable information rate? (2p)

c. For which values of θ_1 is the rate in Part **b** *maximized*? (1p)

d. For which values of θ_1 is the rate in Part **b** *minimized*? (1p)

4 Consider the capacity lower bound for "Downlink with arbitrary precoding vectors" in the table of formulas. An important feature of this capacity bound is that it can be applied along with any channel model, because practical channels are neither deterministic nor Gaussian distributed. (5p)

Suppose $\mathbf{g}_{k'}$ contains independent and identically distributed elements that are $+1$ and -1 with equal probability, for $k' = 1, \ldots, K$.

a. Suppose the base station does not know the channel and selects its precoding vectors as: $\mathbf{a}_k = [1 \ \ldots \ 1]^T$ for $k = 1, \ldots, K$. Compute the downlink capacity lower bound in this case. (1p)

b. Suppose the base station knows the channel and selects $\mathbf{a}_k = \mathbf{g}_k$ for $k = 1, \ldots, K$. Compute the downlink capacity lower bound in this case. (3p)

c. Compare the expression from Part **b** with the case when $\mathbf{g}_{k'}$ contains i.i.d. CN(0,1) elements and $\mathbf{a}_k = \mathbf{g}_k^*$. Which case gives the highest value of the capacity lower bound? (1p)

5 Consider the downlink of a Massive MIMO system that uses maximum ratio processing. There is $K = 1$ user per cell and all users use the same pilot. The channels are modeled by i.i.d. Rayleigh fading and we follow the standard assumptions from "Fundamentals of Massive MIMO". (5p)

a. The base station in cell 1 has M_1 antennas and the base station in cell 2 has M_2 antennas. What is the effective SINRs of the two users? (You don't need to compute the expressions from first principles, but you must carefully explain how and why the new expressions differ from the one in the table of formulas. (1p)

b. What happens to the two SINR expressions as $M_1 \to \infty$ for a fixed value of M_2? Explain the result. (1p)

c. Suppose $M_1 = c_1 M$ and $M_2 = c_2 M$ for two positive constants c_1, c_2. What happens to the two SINR expressions as $M \to \infty$? Explain the result. (1p)

d. Consider the setup in Part **c** and assume that $\rho_{\text{ul}} = \frac{1}{M}$. What happens to the two SINR expressions as $M \to \infty$? Explain the result. (1p)

Table of formulas

Capacity lower bound for the kth terminal in a single-cell system:

Uplink with arbitrary decoding vector:

$$E\left\{\log_2\left(1+\frac{\rho_{\rm ul}\eta_k|\mathbf{a}_k^H\hat{\mathbf{g}}_k|^2}{\mathbf{a}_k^H\left(\sum_{\substack{k'=1\\k'\neq k}}^K\rho_{\rm ul}\eta_{k'}\hat{\mathbf{g}}_{k'}\hat{\mathbf{g}}_{k'}^H+\left(\sum_{k'=1}^K\rho_{\rm ul}\eta_{k'}(\beta_{k'}-\gamma_{k'})+1\right)\mathbf{I}_M\right)\mathbf{a}_k}\right)\right\}$$

Downlink with arbitrary precoding vectors:

$$\log_2\left(1+\frac{\rho_{\rm dl}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}{\sum_{k'=1}^K\rho_{\rm dl}\eta_{k'}E\{|\mathbf{g}_k^T\mathbf{a}_{k'}|^2\}+1-\rho_{\rm dl}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}\right)$$

Effective SINR for the kth terminal in the lth cell in a multi-cell system:

Uplink maximum-ratio:

$$\mathrm{SINR}_{lk}^{\rm mr,ul}=\frac{M\rho_{\rm ul}\gamma_{lk}^l\eta_{lk}}{1+\rho_{\rm ul}\sum_{l'\in\mathcal{P}_l}\sum_{k'=1}^K\beta_{l'k'}^l\eta_{l'k'}+\rho_{\rm ul}\sum_{l'\notin\mathcal{P}_l}\sum_{k'=1}^K\beta_{l'k'}^l\eta_{l'k'}+M\rho_{\rm ul}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{l'k}^l\eta_{l'k}}$$

Downlink maximum-ratio:

$$\mathrm{SINR}_{lk}^{\rm mr,dl}=\frac{M\rho_{\rm dl}\gamma_{lk}^l\eta_{lk}}{1+\rho_{\rm dl}\sum_{l'\in\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^K\eta_{l'k'}\right)+\rho_{\rm dl}\sum_{l'\notin\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^K\eta_{l'k'}\right)+M\rho_{\rm dl}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{lk}^{l'}\eta_{l'k}}$$