Exam in TSKS14 Multiple Antenna Communications

Exam code:	TEN1
Date:	2018-11-01 **Time:** 14:00–18:00
Place:	TER2
Teacher:	Daniel Verenzuela, 013 - 28 26 71, daniel.verenzuela@liu.se Examiner: Emil Björnson, emil.bjornson@liu.se
Visiting exam:	Around 16:00.
Administrator:	Carina Lindström, 013 - 28 44 23, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Pocket calculator with empty memory. Olofsson: Tables and Formulas for Signal Theory.
Number of tasks:	5
Solutions:	Will be published after the exam on the course web page.
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2018-11-15, 12:45–13:00, Emil Björnson's office, Building B, top floor, corridor A between entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

The answer to each question must be supported by an argument or derivation; it is *not* enough to just answer yes/no or to give a number. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Consider the SIMO channel (5p)

$$\boldsymbol{y}[m] = \boldsymbol{g} \cdot x[m] + \boldsymbol{w}[m],$$

where m is the time index, $\{x[m]\}$ is the input sequence with the power limit $E\{|x[m]|^2\} \leq P$, $\{\boldsymbol{y}[m]\}$ is the output sequence, and the noise vector $\{\boldsymbol{w}[m]\}$ has i.i.d. $CN(0, 1)$ entries. The channel $\boldsymbol{g}$ is an $M \times 1$ vector where every element is $+1$ and M denotes the number of receive antennas.

a. What is the capacity of this channel, assuming that the receiver knows $\boldsymbol{g}$? What kind of input distribution achieves the capacity? (2p)

b. Suppose $P = 5$. How many antennas do you need to achieve a capacity of at least 6 bit/s/Hz? (1p)

c. Suppose you have $M = 50$. How large power P is needed to achieve a capacity of 6 bit/s/Hz? (1p)

d. Suppose all entries of $\boldsymbol{g}$ are equal to -1, instead of $+1$. What is the capacity of this channel? Compare the result with Part **a** and explain the intuition behind the result. (1p)

2 Write a text that explains the coherence interval concept for a fellow student. (5p) The text should explain the reason for defining the coherence interval and specifically how it is utilized for transmission in Massive MIMO. Explain and exemplify how one can approximate the size of the coherence interval, based on physical parameters such as the carrier frequency.

3 Consider transmission over a point-to-point MIMO channel where the SNR is $\rho = 2$ and both the transmitter and receiver are equipped with two antennas. (5p)

a. Suppose the channel matrix is

$$\boldsymbol{G} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

Compute the capacity of this channel. (2p)

b. Suppose the channel matrix is

$$\boldsymbol{G} = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}.$$

Compute the capacity of this channel. (2p)

c. Let $\boldsymbol{x} = [x_1 \; x_2]^T$ denote the transmitted signal. How must this signal be distributed to achieve the capacity in Part **b**? (1p)

4 The capacity region of a two-user multiple-access channel (uplink) with perfect channel state information is determined by (5p)

$$\begin{aligned} R_1 &\leq \log_2(1 + P\|\boldsymbol{g}_1\|^2) \\ R_1 + R_2 &\leq \log_2 |\boldsymbol{I}_K + P\boldsymbol{G}^H\boldsymbol{G}| \\ R_2 &\leq \log_2(1 + P\|\boldsymbol{g}_2\|^2). \end{aligned}$$

where $P > 0$ is the SNR and $\boldsymbol{G} = [\boldsymbol{g}_1 \; \boldsymbol{g}_2] \in \mathbb{C}^{M \times K}$.

a. Sketch the capacity region and explain how it is determined by the equations above. (1p)

b. Derive the relation between $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ for which the capacity region is a rectangle. (2p)

c. Suppose $P = 1$, $\boldsymbol{g}_1 = [1, 1, \ldots]^T$, and $\boldsymbol{g}_2 = [1, -1, 1, -1, \ldots]^T$.

For which values of M is the sum rate 10 bit/s/Hz or more? (2p)

5 The effective uplink SINR with maximum-ratio processing is denoted as $\mathrm{SINR}_{lk}^{\mathrm{mr,ul}}$ and is given on the last page for multi-cell scenarios ($L > 1$ cells). An alternative processing scheme is zero-forcing, which was considered in the laboratory exercises. It leads to the alternative effective uplink SINR: (5p)

$$\mathrm{SINR}_{lk}^{\mathrm{zf,ul}} = \frac{(M-K)\rho_{\mathrm{ul}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l}\sum_{k'=1}^{K}(\beta_{l'k'}^{l}-\gamma_{l'k'}^{l})\eta_{l'k'}+\rho_{\mathrm{ul}}\sum_{l'\notin\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+(M-K)\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{l'k}^{l}\eta_{l'k}}.$$

In this problem, you will compare the SINRs achieved with maximum-ratio and zero-forcing in a few different cases.

a. We consider two cases: $\mathcal{P}_l = \{1,\ldots,L\}$ and $\mathcal{P}_l = \{l\}$ for $l = 1,\ldots,L$. Explain the meaning and differences between these two cases. (1p)

b. What is the value of the difference

$$\log_2(1+\mathrm{SINR}_{lk}^{\mathrm{zf,ul}}) - \log_2(1+\mathrm{SINR}_{lk}^{\mathrm{mr,ul}})$$

when $M \to \infty$ if $\mathcal{P}_l = \{1,\ldots,L\}$ and $L > 1$? (1p)

c. What is the value of the difference

$$\log_2(1+\mathrm{SINR}_{lk}^{\mathrm{zf,ul}}) - \log_2(1+\mathrm{SINR}_{lk}^{\mathrm{mr,ul}})$$

when $M \to \infty$ if $\mathcal{P}_l = \{l\}$ and $L > 1$? (1p)

d. Based on results in Part **b** and Part **c**, is zero-forcing or maximum-ratio processing the best scheme as $M \to \infty$? (1p)

e. Suppose there is only one cell. What happens to the difference

$$\log_2(1+\mathrm{SINR}_{1k}^{\mathrm{zf,ul}}) - \log_2(1+\mathrm{SINR}_{1k}^{\mathrm{mr,ul}})$$

when $M \to \infty$ in this case.

Table of formulas

Capacity lower bound for the kth terminal in a single-cell system:

Uplink with arbitrary decoding vector:

$$E\left\{\log_2\left(1+\frac{\rho_{\mathrm{ul}}\eta_k|\mathbf{a}_k^H\hat{\mathbf{g}}_k|^2}{\mathbf{a}_k^H\left(\sum_{\substack{k'=1\\k'\neq k}}^{K}\rho_{\mathrm{ul}}\eta_{k'}\hat{\mathbf{g}}_{k'}\hat{\mathbf{g}}_{k'}^H+\left(\sum_{k'=1}^{K}\rho_{\mathrm{ul}}\eta_{k'}(\beta_{k'}-\gamma_{k'})+1\right)\mathbf{I}_M\right)\mathbf{a}_k}\right)\right\}$$

Downlink with arbitrary precoding vectors:

$$\log_2\left(1+\frac{\rho_{\mathrm{dl}}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}{\sum_{k'=1}^{K}\rho_{\mathrm{dl}}\eta_{k'}E\{|\mathbf{g}_k^T\mathbf{a}_{k'}|^2\}+1-\rho_{\mathrm{dl}}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}\right)$$

Effective SINR for the kth terminal in the lth cell in a multi-cell system:

Uplink maximum-ratio:

$$\mathrm{SINR}_{lk}^{\mathrm{mr,ul}}=\frac{M\rho_{\mathrm{ul}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+\rho_{\mathrm{ul}}\sum_{l'\notin\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+M\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{l'k}^{l}\eta_{l'k}}$$

Downlink maximum-ratio:

$$\mathrm{SINR}_{lk}^{\mathrm{mr,dl}}=\frac{M\rho_{\mathrm{dl}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{dl}}\sum_{l'\in\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^{K}\eta_{l'k'}\right)+\rho_{\mathrm{dl}}\sum_{l'\notin\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^{K}\eta_{l'k'}\right)+M\rho_{\mathrm{dl}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{lk}^{l'}\eta_{l'k}}$$