Exam in TSKS14 Multiple Antenna Communications

Exam code: TEN1

Date: Demo exam 2017 **Time:** Unspecific

Place: Whereever

Teacher: Emil Björnson, tel: 013 - 28 67 32

Visiting exam: One or two visits during the exam

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Department: ISY

Allowed aids: Pocket calculator with empty memory.
Olofsson: Tables and Formulas for Signal Theory.

Number of tasks: 5

Solutions: Will be published after the exam on the course web page.

Result: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

Exam return: Time and place for return unknown

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Consider the channel (5p)

$$y[m] = h \cdot x[m] + w[m],$$

where m is the time index, $\{x[m]\}$ is the input sequence with power P, $\{y[m]\}$ is the output sequence, and $\{w[m]\}$ is i.i.d. $CN(0,1)$ noise. The channel coefficient h is a realization of a random variable which is zero with probability p and one with probability $1-p$. Answer the following questions:

a. What is the outage probability of this channel? Express the answer as a function of the desired rate R. (3p)

b. Suppose we instead have two receive antennas and these observe independent channel realizations, each with the distribution defined above. What is the outage probability for the desired rate R? (2p)

2 Write a text that explains the coherence interval concept for a fellow student. The text should explain the reason for defining coherence interval and specifically how it is utilized for transmission in Massive MIMO. Explain and exemplify how one can approximate the size of the coherence interval, based on physical parameters such as carrier frequency. (5p)

3 Consider the downlink of a Massive MIMO system with M antennas at the base station, two single-antenna user terminals, and $\rho_{\text{ul}} = \rho_{\text{dl}} = 1$. The users have the channel variances $\beta_1 = 1$ and $\beta_2 = 2$. The base station uses maximum-ratio processing and $\tau_p = 2$. Answer the following questions: (5p)

a. The base station needs to select the power-control coefficients η_1, η_2 under the constraint $\eta_1 + \eta_2 \leq 1$. What is the maximum rates that each of the users can get, and which values of the power-control coefficients will give these values? (2p)

b. What is the rate of the two users when the base station allocates power equally to the users. (2p)

c. Sketch the achievable rate region for this setup and mark the operating points that you computed in (a) and (b). (1p)

4 The user terminals in a cell of a Massive MIMO system are assigned mutually orthogonal pilot sequences of length τ_p. (5p)

a. These mutually orthogonal pilot sequences can be collected in a $\tau_p \times K$ matrix $\mathbf{\Psi}$. What mathematical properties will this matrix have? (1p)

b. Now consider a single-cell scenario with $\tau_p = K$. Propose a collection of mutually orthogonal pilot sequences for the user terminals. (1p)

c. Now consider a multi-cell scenario with $\tau_p = 3K$, where $n_{\text{reuse}} = 3$ is the pilot reuse factor. Explain what it means to have a pilot reuse factor. Propose a set of mutually orthogonal pilot sequences that can be used in this scenario and explain how these are assigned to users in different cells. (3p)

5 Consider the uplink of a single-cell Massive MIMO system with M antennas at the base station, three single-antenna user terminals, $\rho_{\text{ul}} = 1$, and $\tau_p = 3$. The users have the channel variances $\beta_1 = 10$, $\beta_2 = 1$, and $\beta_3 = 0.1$. Maximum-ratio processing is being used. (5p)

a. Compute the SINRs that the three users will achieve if they are all transmitting at full power. (2p)

b. Compute the SINRs that the users will achieve under max-min fairness power control. (2p)

c. How many base station antennas are needed in (a) and in (b) to guarantee an SINR of 7 for all users? (1p)

Table of formulas

Capacity lower bound for the kth terminal in a single-cell system:

Uplink with arbitrary decoding vector:

$$E\left\{\log_2\left(1+\frac{\rho_{\mathrm{ul}}\eta_k|\mathbf{a}_k^H\hat{\mathbf{g}}_k|^2}{\mathbf{a}_k^H\left(\sum_{\substack{k'=1\\k'\neq k}}^{K}\rho_{\mathrm{ul}}\eta_{k'}\hat{\mathbf{g}}_{k'}\hat{\mathbf{g}}_{k'}^H+\left(\sum_{k'=1}^{K}\rho_{\mathrm{ul}}\eta_{k'}(\beta_{k'}-\gamma_{k'})+1\right)\mathbf{I}_M\right)\mathbf{a}_k}\right)\right\}$$

Downlink with arbitrary precoding vectors:

$$\log_2\left(1+\frac{\rho_{\mathrm{dl}}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}{\sum_{k'=1}^{K}\rho_{\mathrm{dl}}\eta_{k'}E\{|\mathbf{g}_k^T\mathbf{a}_{k'}|^2\}+1-\rho_{\mathrm{dl}}\eta_k|E\{\mathbf{g}_k^T\mathbf{a}_k\}|^2}\right)$$

Effective SINR for the kth terminal in the lth cell in a multi-cell system:

Uplink maximum-ratio:

$$\frac{M\rho_{\mathrm{ul}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+\rho_{\mathrm{ul}}\sum_{l'\notin\mathcal{P}_l}\sum_{k'=1}^{K}\beta_{l'k'}^{l}\eta_{l'k'}+M\rho_{\mathrm{ul}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{l'k}^{l}\eta_{l'k}}$$

Downlink maximum-ratio:

$$\frac{M\rho_{\mathrm{dl}}\gamma_{lk}^{l}\eta_{lk}}{1+\rho_{\mathrm{dl}}\sum_{l'\in\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^{K}\eta_{l'k'}\right)+\rho_{\mathrm{dl}}\sum_{l'\notin\mathcal{P}_l}\beta_{lk}^{l'}\left(\sum_{k'=1}^{K}\eta_{l'k'}\right)+M\rho_{\mathrm{dl}}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\gamma_{lk}^{l'}\eta_{l'k}}$$