TSKS14 Multiple Antenna Communications

Solutions for the exam Demo exam 2017

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1

We have the channel

$$y[m] = hx[m] + w[m]$$

which, for a given realization h, can support the rate

$$\log_2(1 + P|h|^2),$$

measured in bits per channel use (or bits per complex dimension, or bit/s/Hz). Now, since the channel h is random, so is $|h|^2$:

$$|h|^2 = \begin{cases} 0, & \text{with probability } p \\ 1, & \text{with probability } 1-p. \end{cases}$$

a. The probability of outage for a rate R is given by

$$\mathbb{P}(\log_2(1 + P|h|^2) < R) = \begin{cases} 1, & \text{if } R \geq \log_2(1+P) \\ p, & \text{if } 0 < R < \log_2(1+P). \end{cases}$$

b. Let $\boldsymbol{h} = [h_1, h_2]^{\mathsf{T}}$. We have the channel

$$\boldsymbol{y}[m] = \boldsymbol{h}x[m] + \boldsymbol{w}[m]$$

which, for a given realization $\boldsymbol{h}$, can support the rate

$$\log_2(1 + P\|\boldsymbol{h}\|^2),$$

measured in bits per channel use (or bits per complex dimension, or bit/s/Hz). Then

$$\|\boldsymbol{h}\|^2 = |h_1|^2 + |h_2|^2 = \begin{cases} 0, & \text{with probability } p^2 \\ 1, & \text{with probability } 2(1-p)p \\ 2, & \text{with probability } (1-p)^2. \end{cases}$$

The outage probability is then given by

$$\mathbb{P}(\log_2(1 + P\|\boldsymbol{h}\|^2) < R) = \begin{cases} 1, & \text{if } R \geq \log_2(1+2P) \\ 2(1-p)p + p^2, & \text{if } \log_2(1+P) < R < \log_2(1+2P) \\ p^2, & \text{if } R \leq \log_2(1+P). \end{cases}$$

Answer: See above.

2

A coherence interval is a part of the time-frequency space in which a channel can be considered constant and frequency-flat, so that the channel can be described by constant multiplication with a scalar/vector/matrix. The number of symbols in a coherence interval is $\tau_c = T_\mathrm{c} B_\mathrm{c}$, where T_c is the coherence time and B_c is the coherence bandwidth.

The coherence time is affected by how quickly things around the transmitter and the received move (or how quickly the transceivers move). The quicker they more, the faster the channel will change. A rule of thumb is that T_c is in the order of $\frac{\lambda}{2v}$, where λ is the wavelength of the carrier and v is the speed of fastest moving object in the vicinity of the transceivers.

The coherence bandwidth is affected by the time difference between the received signal components in a multipath environment. The larger the difference is between the shortest and the longest path (measured in time by the delay spread T_d), the smaller is the coherence bandwidth. A rule of thumb is that B_c is in the order of c/T_d, where c is the speed of light and T_d is the delay spread.

The coherence interval concept is important in massive MIMO because, in order to use the many antennas, the base station first estimates the channel. This channel estimate is only valid for τ_c samples (channel uses), and then the base station has to estimate the channel again. The finite coherence interval dimensionality is the reason why massive MIMO need TDD operation in order to handle a large number of antennas, since in TDD the number of pilots is proportional to the number of users. Pilots, uplink data, and downlink data must all fit into one coherence interval.

Answer: See above.

3

Let's consider the effective downlink SINR of terminal k:

$$\mathrm{SINR}_k = \frac{M \rho_\mathrm{dl} \gamma_k \eta_k}{1 + \rho_\mathrm{dl} \beta_k \sum_{k'=1}^{K} \eta_{k'}}. \tag{1}$$

This, as we have shown in the tutorials, is an increasing function of η_k. Moreover, note that

$$\gamma_k = \frac{\rho_\mathrm{ul} \tau_\mathrm{p} \beta_k^2}{1 + \rho_\mathrm{ul} \tau_\mathrm{p} \beta_k},$$

so $\gamma_1 = 2/3$ and $\gamma_2 = 8/5$.

a. The maximum SINR for user k is achieved when $\eta_k = 1$. Plugging this, together with the given values for the other parameters, into (1) gives the maximum rates

$$\log_2\left(1 + \frac{M}{3}\right)$$

and

$$\log_2\left(1 + \frac{8M}{15}\right)$$

for user 1 and 2, respectively.

b. Equal power to both users implies $\eta_1 = \eta_2 = 1/2$, which gives the rates

$$\log_2\left(1+\frac{M}{6}\right)$$

and

$$\log_2\left(1+\frac{4M}{15}\right)$$

for user 1 and 2, respectively.

c. We sketch the rate region for the two users (we used $M = 10$ to compute the curve, while different M will give a slightly different shape)

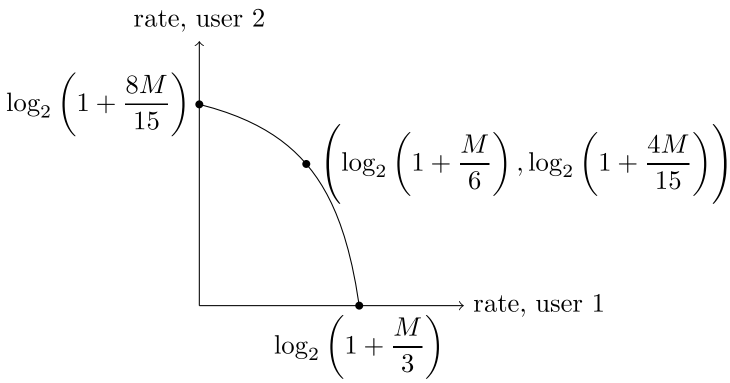

Figure 1: Rate region for the two users in problem 3.

Answer: See above

4

a. Orthogonality between two different pilot sequences means that their inner product is zero. With $\boldsymbol{\Psi} = [\boldsymbol{\psi}_1, \ldots, \boldsymbol{\psi}_K]$, $\boldsymbol{\psi}_k^{\mathsf{H}}\boldsymbol{\psi}_{k'} = 0$ and $\boldsymbol{\psi}_k^{\mathsf{H}}\boldsymbol{\psi}_k = \alpha_k > 0$, we have

$$\boldsymbol{\Psi}^{\mathsf{H}}\boldsymbol{\Psi} = \alpha \boldsymbol{I}_K.$$

This is assuming that $\tau_{\mathrm{p}} \geq K$ and α_k take any strictly positive value, although $\alpha_k = 1$ is assumed in the course book. Note that the pilot sequences can be orthogonal, but not have the same magnitude, in which case $\boldsymbol{\Psi}^{\mathsf{H}}\boldsymbol{\Psi}$ will be a diagonal matrix, with different, non-zero element along its diagonal.

b. We are looking for a square matrix with mutually orthogonal columns. The simplest example is the identity matrix $\boldsymbol{I}_K$. (Satisfies the property with $\alpha_k = 1$ for $k = 1, \ldots, K$.)

c. A pilot reuse factor is used to decrease the pilot contamination between different cells. By making pilot sequences longer, there are more sequences to choose from. Cells very close to each other can then choose different sets of orthogonal sequences to avoid interfering with each other. A pilot reuse factor of 3 means that the cells are divided into three disjoint groups, each with its own set of mutually orthogonal pilots. These pilot sequences are orthogonal within a cell and between different groups.

The identity matrix $\boldsymbol{I}_{3K}$ is a set of $3K$ mutually orthogonal columns (pilot sequences). Let $\boldsymbol{\Psi}_l$ the pilot matrix for group l (l=1,2,3). By letting

$$[\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \boldsymbol{\Psi}_3] = \boldsymbol{I}_{3K},$$

each group has a set of K sequences, mutually orthogonal, and orthogonal to all sequences in the other two groups. Each cell in group l now assigns each of the K users a different column in $\boldsymbol{\Psi}_l$.

Answer: See above.

5

The SINR for user k in the uplink is given by

$$\mathrm{SINR}_k = \frac{M\rho_{\mathrm{ul}}\gamma_k\eta_k}{1+\rho_{\mathrm{ul}}\sum_{k'=1}^{K}\beta_{k'}\eta_{k'}},$$

where

$$\gamma_k = \frac{\rho_{\mathrm{ul}}\tau_{\mathrm{p}}\beta_k^2}{1+\rho_{\mathrm{ul}}\tau_{\mathrm{p}}\beta_k} = \begin{cases} \frac{300}{31}, & k=1 \\ \frac{3}{4}, & k=2 \\ \frac{3}{130}, & k=3 \end{cases}$$

a. Plugging all the numbers into the SINR expression gives $\mathrm{SINR}_1 = \frac{300M}{31\cdot 12.1}$, $\mathrm{SINR}_2 = \frac{3M}{4\cdot 12.1}$, and $\mathrm{SINR}_3 = \frac{3M}{130\cdot 12.1}$

b. Here, $\eta_k = \frac{\min_{k'}(\gamma_{k'})}{\gamma_k}$ and $\mathrm{SINR}_k = \mathrm{SINR}_{k'}$. Since all SINRs are equal, we look at user 3, since $\eta_3 = 1$.

$$\mathrm{SINR} = \frac{M\rho\gamma_3}{1+\rho\sum_{k'=1}^{3}\frac{\beta_{k'}\gamma_3}{\gamma_{k'}}} = \frac{3}{150.1}M$$

c. To have all SINRs above 7 we would need

$$\frac{3M}{130\cdot 12.1} > 7 \Rightarrow M > 3670$$

and

$$\frac{3M}{150.1} > 7 \Rightarrow M > 350$$

for (a) and (b), respectively.

Answer: See above.