Exam in TSKS04 Digital Communication Continuation Course

Exam code: TEN1

Date: 2017-06-05 **Time:** 14:00–18:00

Place: G35

Teacher: Emil Björnson, tel: 013-286732

Visiting exam: Around 15 and 16:30

Administrator: Carina Lindström, 013-284423, carina.e.lindstrom@liu.se

Department: ISY

Allowed aids: Olofsson: *Tables and Formulas for Signal Theory*
Upamanyo Madhow: *Fundamentals of Digital Communication*, Cambridge University Press, 2008.

Number of tasks: 5

Solutions: Will be published within a week after the exam at http://www.commsys.isy.liu.se/TSKS04

Result: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

Exam return: 2017-06-16, 12:45–13.00, in the office of Emil Björnson, Building B, Corridor A, between Entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

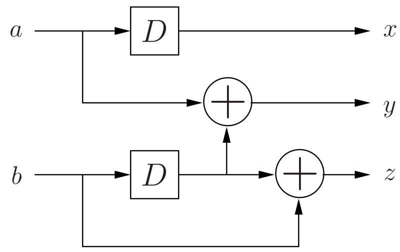
Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Consider the following convolutional encoder. (5 p)



a. Draw the state diagram of the encoder. Indicate both input and output on all edges.

b. Draw one section of the trellis of the encoder. Indicate both input and output on all edges.

c. Determine the free distance of the code.

2 Consider digitally modulated communication with symbol rate $1/T$. The transmit filter, channel and receive filter have the impulse responses (5 p)

$$g_{\mathrm{TX}}(t) = g_{\mathrm{C}}(t) = I_{\{0 \le t < T\}}(t), \qquad g_{\mathrm{RX}}(t) = \begin{cases} t, & 0 \le t < T/2, \\ T - t, & T/2 \le t < T, \\ 0, & \text{elsewhere}, \end{cases}$$

respectively. The channel also adds white Gaussian Noise. Let $z[k]$ denote the receive filter output sampled at time instance $kT_{\mathrm{s}} + \tau$, where T_{s} is a sampling interval to be chosen and τ is some time-shift, also to be chosen.

a. Show that ML sequence detection using the samples $\{z[k]\}$ is possible, given an appropriate choice of T_{s} and τ. Specify the corresponding choice of T_{s} and τ. *If ML sequence detection is not possible for any choice of T_{s} and τ, then show that that is the case.*

b. How many states are needed in the trellis for implementing ML sequence detection using the Viterbi algorithm? *In the case that ML detection is not possible, suggest an alternative choice of sender and/or receiver filter to make ML sequence detection possible.*

Note: There may very well be more than one solution to this problem. It is enough to find one of them.

3 Consider a linearly modulated communication system, where we model the received samples $Y[k]$ after demodulation as (5 p)

$$Y[k] = b[k]e^{j2\pi\,\Delta f\,kT} + W[k], \qquad k = 1, \ldots, K,$$

where $b[k]$ are samples from a known complex-valued training sequence, Δf is an unknown frequency offset, T is the symbol time and $W[k]$ is discrete-time complex WGN with variance $\sigma^2 = N_0/2$ per dimension. We wish to obtain an ML estimate of the frequency offset. Therefore, determine a suitable cost function for this situation.

4 Consider the two generator matrices (5 p)

$$G_1(D) = \begin{pmatrix} 1 & 0 & 1+D+D^2 \\ 0 & 1 & 1+D \end{pmatrix},$$

$$G_2(D) = \begin{pmatrix} 1 & 1+D & D \\ 1 & D & 1 \end{pmatrix}.$$

a. Show that both matrices generate the same convolutional code.

b. Is any of the two generator matrices catastrophic?

5 Let the input to a digital linear modulator consist of independent, equally probable bits. Consider an on-off keying signal constellation with the signal variance P. Determine the power-spectral density when the basis function is (5 p)

$$\phi_1(t) = \cos(2\pi f_c t), \quad 0 \le t < T,$$

where $2f_cT$ is a positive integer.