LINKÖPINGS UNIVERSITET

Exam in TSKS04 Digital Communication Continuation Course

Exam code:	TEN1
Date:	2017-03-16 **Time:** 08:00–12:00
Place:	TER1
Teacher:	Mikael Olofsson, tel: 013-281343. (Examiner: Emil Björnson)
Visiting exam:	9 and 11
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* Upamanyo Madhow: *Fundamentals of Digital Communication*, Cambridge University Press, 2008.
Number of tasks:	5
Solutions:	Will be published within a week after the exam at http://www.commsys.isy.liu.se/TSKS04
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2017-04-03, 12:30–13.00, in the office of Emil Björnson, Building B, Corridor A, between Entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Consider a pulse-amplitude modulated signal (5 p)

$$Y(t) = \sum_{n=-\infty}^{\infty} S[n]\phi(t - nT - \Psi)$$

where the basis function is

$$\phi(t) = \cos(2\pi f_c t), \quad -T/2 \leq t < T/2,$$

and Ψ is is a random delay uniformly distributed between 0 and T. The symbols $S[n]$ originate from a 4-ASK constellation with $\pm\sqrt{E_{\max}}$ being the two outmost constellation points. The constellation points are equally probable and subsequent symbols are independent.

Compute the power-spectral density of $Y(t)$.

2 Consider a convolutional encoder with one input and two outputs, with the respective generator polynomials $g_1(D) = 1 + D^2$ and $g_2(D) = 1 + D + D^2$. (5 p)

a. Draw a trellis or a state diagram that describe the possible state transitions in this encoder. Each transition should be labeled with the corresponding input and output bits.

b. The encoder is used for an unknown input sequence of four bits, followed by two zeros that terminate the convolutional code. Suppose the output signals are $y_1(D) = D + D^2 + D^3 + D^4$ and $y_2(D) = 1 + D + D^3 + D^4$. Use the Viterbi algorithm to compute the ML estimate of the four input bits.

3 Solve Problem 4.11 on Page 197 in Madhow. (5 p)

4 Consider a channel with the sampled output signal (5 p)

$$y[n] = \sum_{j=0}^{L-1} s[n-j]h[j] + w[n],$$

where $s[n]$ are the input symbols and $h[0], \ldots, h[L-1]$ are *unknown* channel coefficients. The noise is white, Gaussian, has zero mean and variance 1.

To mitigate inter-symbol interference when communicating over this channel, it is highly desired to estimate the channel coefficients. Suppose we observe $y[n]$ for $n = 0, \ldots, L-1$ and that we know the following input symbols:

$$s[k] = \begin{cases} 0 & \text{for } k < 0, \\ 1 & \text{for } k = 0, \ldots, K-1. \end{cases}$$

a. Compute the Cramer-Rao lower bound (CRB) for estimation of $h[0], \ldots, h[L-1]$, based on $y[0], \ldots, y[K-1]$.

b. Compute an estimator that achieves the CRB, or prove that such an estimator does not exist.

5 Consider a channel with the sampled output signal (5 p)

$$y[n] = \sum_{j=0}^{L-1} s[n-j]h[j] + w[n],$$

where $s[n]$ are the input symbols and $h[0], \ldots, h[L-1]$ are *known* channel coefficients. The noise is white, Gaussian, has zero mean and variance 1.

This channel causes inter-symbol interference, which needs to be mitigated before symbol detection. Suppose $s[k] = 0$ for $k < 0$ and that we would like to detect $s[0], \ldots, s[N-1]$.

a. Derive a vector/matrix expression that shows how the input symbols $s[0], \ldots, s[N-1]$ and output $y[0], \ldots, y[N-1]$ are related.

b. Derive an expression for a linear equalizer that removes the inter-symbol interference, but not the desired symbols.

c. Exemplify how your linear equalizer works in the case of $L = 2$ and $N = 2$. (You can select any non-zero values of the known coefficients.)