LINKÖPINGS UNIVERSITET

Exam in TSKS04 Digital Communication Continuation Course

Exam code:	TEN1
Date:	2016-06-07 **Time:** 14:00–18:00
Place:	P36
Teacher:	Emil Björnson, tel: 013-286732
Visiting exam:	15 and 16
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* Upamanyo Madhow: *Fundamentals of Digital Communication*, Cambridge University Press, 2008.
Number of tasks:	5
Solutions:	Will be published within some days after the exam at http://www.commsys.isy.liu.se/TSKS04
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2016-06-21, 11.00–11.30, in the office of Emil Björnson, Building B, Corridor A, between Entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Consider a pulse-amplitude modulated signal (5 p)

$$Y(t) = \sum_n S[n]\phi(t - nT - \Psi)$$

where the basis function is

$$\phi(t) = \sin(2\pi f_c t), \quad -T/2 \le t < T/2,$$

and Ψ is is a random delay uniformly distributed between 0 and T. The symbols $S[n]$ originate from an on-off keying constellation where the points 0 and $\sqrt{E}$ are equally probable. Subsequent symbols are independent.

Determine the power-spectral density of $Y(t)$.

2 Consider the generator matrix (5 p)

$$G(D) = \begin{pmatrix} 1 + D^2 & D^2 & \frac{1}{1+D^2} \end{pmatrix}$$

of a convolutional code.

a. Draw an encoder for this code.

b. Compute the codeword associated with the input sequence $u = (1\,0\,1\,0\,0\,\ldots)$.

c. Can this code generate a codeword with *finite* Hamming weight for an input sequence with *infinite* Hamming weight? If yes, provide an example. If no, prove that this is not possible.

d. Can this code generate a codeword with *infinite* Hamming weight for an input sequence with *finite* Hamming weight? If yes, provide an example. If no, prove that this is not possible.

3 The gain g of a communication channel is to be estimated at the receiver. Suppose that a training sequence is transmitted such that the receiver observes (5 p)

$$y[1] = g + w[1]$$
$$y[2] = g + w[2]$$

where $w[1], w[2]$ are realizations of the measurement noise. These are real-valued jointly Gaussian random variables with zero mean and the covariance matrix

$$\mathbf{C} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}.$$

Note that the two noise realizations are correlated.

a. Compute the log-likelihood function $\ln p(\mathbf{y}|g)$, where $\mathbf{y} = (y[1], y[2])^T$.

b. Compute the Cramer-Rao lower bound for estimation of g based on the observation $\mathbf{y}$.

c. Explain and prove what happens to the Cramer-Rao lower bound when $\rho \to 1$.

4 Consider information symbols $b[n]$ from a BPSK constellation, where $b[n] = -\sqrt{E}$ or $b[n] = +\sqrt{E}$ with equal probability for $n = 1, \ldots, 3$. (5 p)

These symbols are transmitted over a channel that unfortunately causes inter-symbol interference. The received signal is given by

$$y(t) = \sum_{n=-\infty}^{\infty} b[n]p(t-n),$$

where $p(t)$ is the impulse response and it has the sampled autocorrelation sequence

$$h[m] = \int p(t)p^*(t-mT)dt = \begin{cases} -2, & m = 0 \\ 2, & m = \pm 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Assume that $b[n] = -\sqrt{E}$ for all $n \leq 0$. Select an appropriate algorithm and find the ML estimates of $b[1], b[2], b[3]$, using the matched filter outputs (defined in (5.3) in Madhow) $z[1] = 0.8\sqrt{E}$, $z[2] = 0$, $z[3] = 0.2\sqrt{E}$.

5 Solve Problem 4.12ab on page 197 in Madhow. (5 p)