Exam in TSKS04 Digital Communication Continuation Course

Exam code:	TEN1
Date:	2016-03-16 **Time:** 08:00–12:00
Place:	T2 and U3
Teacher:	Emil Björnson, tel: 013-286732
Visiting exam:	9 and 11
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* Upamanyo Madhow: *Fundamentals of Digital Communication*, Cambridge University Press, 2008.
Number of tasks:	5
Solutions:	Will be published within some days after the exam at http://www.commsys.isy.liu.se/TSKS04
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2016-04-04, 12.30–13.00, in the Hamming room close to the office of Emil Björnson, Building B, Corridor A, between Entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Orthogonal frequency-division multiplexing (OFDM) is a digital modulation scheme that is used in many cable and wireless standards. An OFDM signal can be expressed as (5 p)

$$S(t) = \sum_{n=-\infty}^{\infty} \sum_{i=0}^{N-1} \Big(C_i[n]\phi_i(t - nT - \Psi) - S_i[n]\gamma_i(t - nT - \Psi) \Big)$$

where T is the symbol time and Ψ is a random delay uniformly distributed between 0 and T.

Suppose the information sequences $C_i[n]$ and $S_i[n]$ contain independent symbols from a BPSK constellation with variance E. Let the basis functions be

$$\phi_i(t) = \cos\left(2\pi\left(f_c + \frac{i}{T}\right)t\right), \quad \text{for} \quad 0 \le t < T$$

$$\gamma_i(t) = \sin\left(2\pi\left(f_c + \frac{i}{T}\right)t\right), \quad \text{for} \quad 0 \le t < T,$$

and zero elsewhere, for some carrier frequency f_c such that $2f_cT$ is a positive integer.

a. Determine the power-spectral density of the modulated signal $S(t)$.

b. Give a reasonable approximation of the bandwidth of $S(t)$ and give a theoretical motivation to your approximation.

2 Consider the generator matrix (5 p)

$$G(D) = \begin{pmatrix} 1 + D^2 & D + D^2 \end{pmatrix}$$

of a convolutional code.

a. Draw a shift-register implementation of an encoder for this code.

b. Draw the state diagram of the code.

c. Compute the free distance of the code.

3 The receiver in a communication system needs to know the carrier frequency f_0 that the transmitter is using. Suppose that the receiver observes (5 p)

$$y[k] = A\cos(2\pi f_0 k) + w[k]$$

for $k = 1, \ldots, K$, where $w[1], \ldots, w[K]$ are realizations of the measurement noise and $A > 0$ is a known constant amplitude. The measurement noise is jointly Gaussian distributed with mean value 0 and covariance matrix $\sigma^2 \mathbf{I}$.

a. Compute the log-likelihood function $\ln p(\mathbf{y}|f_0)$, where $\mathbf{y} = (y[1], \ldots, y[K])^T$.

b. Compute the Cramer-Rao lower bound for estimation of f_0 based on the observation $\mathbf{y}$.

c. Explain and prove what happens to the Cramer-Rao lower bound as the number of observations $K \to \infty$. Does the behavior depend on the value of f_0?

4 Consider information symbols $b[n]$ from an on-off keying contellation, where $b[n] = 0$ or $b[n] = 1$ with equal probability for $n = 1, \ldots, 4$. (5 p)

These symbols are transmitted over a channel that unfortunately causes inter-symbol interference. The received signal is given by

$$y(t) = \sum_{n=-\infty}^{\infty} b[n]p(t-n),$$

where $p(t)$ is the impulse response and it has the sampled autocorrelation sequence

$$h[m] = \int p(t)p^*(t-mT)dt = \begin{cases} 2, & m = 0 \\ -1, & m = \pm 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Assume that $b[n] = 0$ for all $n \leq 0$. Select an appropriate algorithm and find the ML estimates of $b[1], b[2], b[3], b[4]$, using the matched filter outputs (defined in (5.3) in Madhow) $z[1] = 0.7$, $z[2] = 1.2$, $z[3] = -0.4$, $z[4] = 0.3$.

5 Consider the 3×1 received vector (5 p)

$$\mathbf{r}[n] = b[n] \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}_{=\mathbf{u}_0} + b[n-1] \underbrace{\begin{pmatrix} 1.5 \\ -1.5 \\ -1 \end{pmatrix}}_{=\mathbf{u}_{-1}} + \mathbf{w}[n],$$

where $\mathbf{w}[n]$ is zero mean proper complex Gaussian noise with covariance matrix

$$\mathbf{C}_w = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

This received vector is used to make a decision on $b[n]$, under interference from $b[n-1]$.

a. Compute the linear zero-forcing equalizer.

b. Compute a correlator vector $\mathbf{c}$ that minimizes the noise variance $\mathbf{c}^H \mathbf{C}_w \mathbf{c}$ subject to the requirement $\mathbf{c}^H \mathbf{u}_0 = 1$.

c. Does there exist a correlator $\mathbf{c}$ for this problem setup that removes both inter-symbol interference and noise, while $\mathbf{c}^H \mathbf{u}_0 = 1$?