INSTITUTE OF TECHNOLOGY
LINKÖPINGS UNIVERSITET

Exam in TSKS04 Digital Communication Continuation Course

Exam code: TEN1

Date: 2015-08-21 **Time:** 08:00–12:00

Place: TER1

Teacher: Emil Björnson, tel: 013-286732

Visiting exam: 8:45 and 11

Administrator: Carina Lindström, 013-284423, carina.e.lindstrom@liu.se

Department: ISY

Allowed aids: Olofsson: *Tables and Formulas for Signal Theory*
Upamanyo Madhow: *Fundamentals of Digital Communication*, Cambridge University Press, 2008.

Number of tasks: 5

Solutions: Will be published within some days after the exam at
http://www.commsys.isy.liu.se/TSKS04

Result: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

Exam return: 2015-09-08, 12.30–13.00, in the office of Emil Björnson, Building B, Corridor A, between Entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Let the input to a digital linear modulator consist of independent, equally probable bits. Consider a signal constellation where the symbols are 0, $\sqrt{2}A$, and $2A$ with equal probability. (5 p)

a. Determine the mean and variance of the signal constellation.

b. Determine the power-spectral density of the modulated signal when the basis function is

$$\phi_1(t) = \sin(2\pi f_c t), \quad 0 \leq t < T,$$

where $2f_cT$ is a positive integer.

2 Consider the generator matrix (5 p)

$$G(D) = \begin{pmatrix} 1 + D^2 & D & \frac{1}{1+D^2} \end{pmatrix}$$

of a convolutional code.

a. Draw an encoder for this code.

b. What is the rate of the code?

c. Prove whether or not this encoder is catastrophic.

3 Suppose that we observe (5 p)

$$y = s + n,$$

where s is a desired signal and n is noise.

We usually assume that s and n are independent random variables. However, in practice, y is often a voltage measured in a circuit at the receiver, where s and n are two other (input) voltages in the same circuit. Hence, s and n can interact with each other; in particular, the variance of n may depend on the realizations of s.

Suppose that n is Gaussian distributed with mean 0 and variance $\sigma^2|s|$, which apparently depends on s. Consider a signal s that takes the values A, $2A$, and $3A$ with equal probability.

Find and sketch the ML decision regions for s given the observation y. Please make sure to provide expressions (or equations, if the expressions are hard to simplify) for all the ML decision boundaries.

4 We would like to compute ML estimates of the information symbols $b[n]$ for $n = 1, \ldots, 4$, assuming that $b[n] = -1$ for all $n \leq 0$. (5 p)

The received signal is given by

$$y(t) = \sum_{n=-\infty}^{\infty} b[n]p(t-n),$$

and we know that the pulse p has the sampled autocorrelation sequence

$$h[m] = \int p(t)p^*(t-mT)dt = \begin{cases} 0, & m = 0 \\ -1, & m = \pm 1 \\ 1, & m = \pm 2 \\ 0, & \text{elsewhere.} \end{cases}$$

Moreover, the matched filter outputs (defined as in (5.3)) are $z[1] = 1$, $z[2] = -3$, $z[3] = 0$, $z[4] = -2$. Select an appropriate algorithm and find the ML estimates of $b[1], b[2], b[3], b[4]$.

5 Solve Problem 7.7 on page 371 in Madhow. (5 p)