INSTITUTE OF TECHNOLOGY
LINKÖPINGS UNIVERSITET

Exam in TSKS04 Digital Communication Continuation Course

Exam code:	TEN1
Date:	2015-03-16 **Time:** 8:00–12:00
Place:	U10
Teacher:	Mikael Olofsson, tel: 281343 (examiner and creater: Emil Björnson)
Visiting exam:	9 and 11
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* Upamanyo Madhow: *Fundamentals of Digital Communication*, Cambridge University Press, 2008.
Number of tasks:	5
Solutions:	Will be published within three days after the exam at http://www.commsys.isy.liu.se/TSKS04
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2015-03-30, 12.30–13.00, Filtret, Building B, ground floor, corridor D, close to Entrance 29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of five problems. You can get up to five points from each problem. Thus, at most 25 points are available. Grade limits:

- Grade three: 12 points,
- Grade four: 16 points,
- Grade five: 20 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

1 Consider a 4-PSK signal constellation where all signals are equally probable, the signal variance is P, and subsequent symbols are independent. Determine the power-spectral density when the basis functions are (5 p)

$$\begin{aligned} \phi_1(t) &= \sin(2\pi f_c t), \quad 0 \le t < T, \\ \phi_2(t) &= \cos(2\pi f_c t), \quad 0 \le t < T, \end{aligned}$$

where $2f_cT$ is a positive integer.

2 Answer the following questions on linear equalization (a convincing motivation is needed): (5 p)

a. The ZF equalizer, if it exists, is given by (5.27) in Madhow. Give a concrete example when ZF does not exist.

b. Under which conditions can the ZF equalizer be considered optimal?

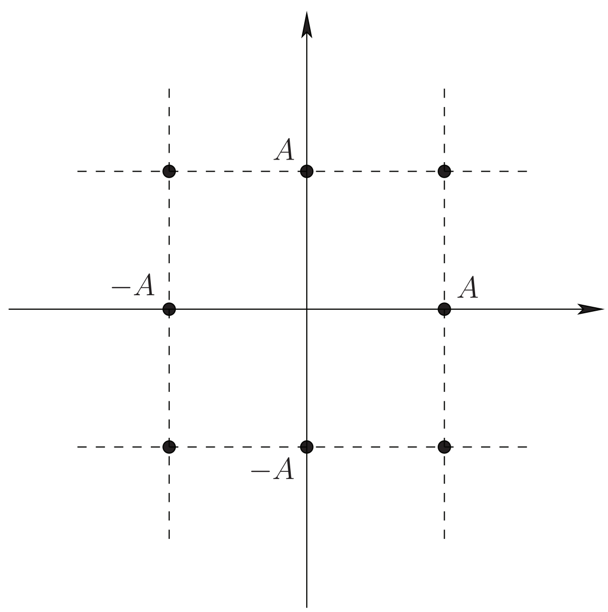
c. What is the smallest possible noise enhancement factor?

d. Under which conditions are the ZF and MMSE equalizers equal?

3 Consider a random two-dimensional vector (5 p)

$$\mathbf{y} = \mathbf{s} + \mathbf{n}$$

where $\mathbf{n}$ is AWGN with zero mean and variance σ^2. Suppose that $\mathbf{s}$ originates from the following constellation:



These eight signal points are equally probable.

a. Formulate the hypotheses tests for each of the signal points. Simplify the expressions as far as possible.

b. Sketch the ML decision regions.

c. Compute the log-likehoods when $\mathbf{y} = \begin{pmatrix} \frac{A}{3} \\ -\frac{6A}{5} \end{pmatrix}$.

4 Consider a channel where the received signal is given by (5 p)

$$y(t) = \sum_{n=-\infty}^{\infty} b[n]p(t-n),$$

where $\{b[n]\}$ is a symbol sequence from a BPSK modulation (i.e., $b[n] \in \{-1, +1\}$). The pulse p is given by

$$p(t) = \begin{cases} 2, & 1 \le t < 2 \\ \frac{1}{2}, & 2 \le t < 3 \\ -\frac{1}{2}, & 3 \le t < 4 \\ 0, & \text{elsewhere.} \end{cases}$$

a. Compute the sampled autocorrelation sequence of the pulse p.

b. What is the memory L of this channel?

c. Suppose that $b[n] = +1$ for all $n \le 0$. Use the Viterbi algorithm to compute ML estimates of the received symbols $b[n]$ for $n = 1, \dots, 4$, assuming that the matched filter outputs are $z[1] = 1.25$, $z[2] = 3$, $z[3] = 0$, $z[4] = -1.5$.

5 Consider the generator matrix (5 p)

$$G(D) = \begin{pmatrix} 1 + D^2 & D & \frac{1}{1+D} \end{pmatrix}$$

of a convolutional code.

a. Draw an encoder for this code.

b. Compute the codeword associated with the input sequence $u = (0\,1\,0\,1\,0\,0\,\dots)$

c. Can this code generate a codeword with infinite Hamming weight for an input sequence with finite Hamming weight? If yes, provide an example. If no, prove that this is not possible.