

Exam in TSKS04 Digital Communication

Continuation Course

Exam Code:	TEN1
Date & Time:	14:00 - 18:00, 3 June, 2013
Place:	G34
Teacher:	Mikael Olofsson, tel: 281343
Exam Visit:	Approximately 15 and 17
Department:	ISY
Allowed aids:	Olofsson: Tables and Formulas for Signal Theory U. Madhow: Fundamentals of Digital Communication
Grade Translation:	Grades 3,4, and 5, are translated to ECTS C,B, and A
Solutions:	Next day after the exam on the course web page
Grading:	Maximum points: 100, Pass: > 40, Grade 3: 41 – 57, Grade 4: 58 – 74, Grade 5: 75 – 100
Important Instructions:	All answers must be given in English Please write legibly since partial points will be awarded for each question even if the final answer is incorrect



INSTITUTE OF TECHNOLOGY
LINKÖPING UNIVERSITY

1) **Complex baseband representation for down conversion with a phase offset** (20 points)

Let the transmitted passband signal be given by $x_p(t) = \text{Re}(x(t)e^{j2\pi f_c t})$ where f_c is the carrier frequency and $x(t)$ is the complex baseband representation of the transmitted signal. Further, $x(t)$ has its Fourier transform band-limited to $[-W, W]$ (assume that $f_c > W$).

Consider an ideal channel where the transmitted signal is received as it is, i.e., the received passband signal is given by $y_p(t) = x_p(t)$. However, the local oscillator at the receiver has a phase offset of θ radians with respect to the phase of the local oscillator used for up conversion in the transmitter.

The output of the I branch ($z^I(t)$) in the receiver is given by

$$\begin{aligned} y_p^I(t) &= y_p(t) \cos(2\pi f_c t + \theta) \\ z^I(t) &= g(t) \star y_p^I(t) \end{aligned} \quad (1)$$

where \star denotes convolution and $g(t) = 2W \text{sinc}(2Wt)$. Similarly the Q branch output is given by

$$\begin{aligned} y_p^Q(t) &= -y_p(t) \sin(2\pi f_c t + \theta) \\ z^Q(t) &= g(t) \star y_p^Q(t). \end{aligned} \quad (2)$$

Show that

$$z(t) \triangleq z^I(t) + jz^Q(t) = \frac{1}{2}x(t) e^{-j\theta} \quad (3)$$

which corresponds to a rotation by $-\theta$ radians.

$$(j \triangleq \sqrt{-1} \text{ and } \text{sinc}(x) \triangleq \frac{\sin(\pi x)}{\pi x})$$

2) **(Modulation)** (20 points)

a) **(Tone spacing in Multicarrier communication)** (10 points)

Consider two real-valued passband waveforms

$$\begin{aligned} s_0(t) &= \cos(2\pi f_0 t), \quad 0 \leq t \leq T \\ s_1(t) &= \cos(2\pi f_1 t), \quad 0 \leq t \leq T \end{aligned} \quad (4)$$

where $f_1 > f_0 \gg 1/T$ (lets assume $f_0, f_1 > 10^9$ Hz and $T = 10$ milli second.) The pulses will be said to be orthogonal if $\int_0^T s_0(t)s_1(t) dt = 0$.

Show that the minimum frequency separation such that the pulses are orthogonal is $f_1 - f_0 = \frac{1}{2T}$.

b) **(Spectral efficiency of modulation schemes)** (10 points)

Consider the following two transmission schemes.

Scheme-I: The complex baseband signal is given by

$$x(t) = \sum_{k=0}^{M-1} x[k] \text{sinc}\left(\frac{t}{T} - k\right) \quad 0 \leq t \leq MT \quad (5)$$

where $x[k] \in +1, -1$ are the information carrying BPSK symbols, and M is an integer. Assume that the information symbols are independent of each other.

Scheme-II: Consider M mutually orthogonal complex baseband waveforms $\{\phi_1(t), \dots, \phi_M(t)\}$, i.e., $\int \phi_i(t)\phi_k^*(t) dt = 0$ if $i \neq k$. Also assume that these waveforms are approximately time limited to $[0, MT]$ and bandlimited to $[-1/2T, 1/2T]$. There are totally M possible messages that can be communicated, and to communicate the m -th message the transmitter sends the waveform $\phi_m(t)$.

Which scheme is more spectral efficient (bandwidth efficient) and why ?

3) **(Detection in ISI channels)** (20 points)

Consider an ISI channel with additive noise. The complex baseband discrete-time input and output are related by

$$Y[n] = \sum_{k=0}^1 h[k]X[n-k] + W[n] \quad (6)$$

where $Y[n]$ and $X[k]$ are the n -th discrete time output sample and the k -th information symbol respectively. $h[k]$, $k = 0, 1$ is the impulse response of the channel filter. Let $h[0] = 2$ and $h[1] = -j$.

We will assume that the sequence of random variables $\{W[n]\}$ are independent and identically distributed (i.i.d.). Assume $W[n] = W^I[n] + W^Q[n]$ to be complex valued with independent and identically distributed real and imaginary components. Let the real component i.e., $W^I[n]$ have a continuous probability density function (p.d.f.) given by $f_W(w) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-w^2/(2\sigma^2)}$.

Let the input information symbols belong to the BPSK alphabet $\mathcal{A} = \{+1, -1\}$, i.e., $X[k] \in \mathcal{A}$. Also, let the input be time limited, i.e., $X[k] = 0$ for $k < 0$ and $k > 2$. Further, the input information symbols $\{x[k]\}$ are independent of additive noise.

Derive an expression for the maximum likelihood detector (MLD) of the three input BPSK symbols given the received discrete-time samples $y[n]$, $n = 0, 1, 2, 3$.

What is the output of the MLD if $y[0] = j, y[1] = 1 - j, y[2] = 2 + j, y[3] = 1$?

4) **(Noncoherent Communication)** (25 points)

Consider a bandlimited passband information bearing transmit signal given by

$$X_p(t) = \text{Re}\left(X(t)e^{j(2\pi f_c t + \theta)}\right), \quad t \in [0, T_0] \quad (7)$$

where f_c is the carrier frequency and θ is the phase of the local oscillator (LO) in the transmitter. $X(t)$ is the complex baseband information bearing signal which is bandlimited to $[-W, W]$. Assume that $f_c \gg W$ and also that $f_c T_0 \gg 1$.

Let the received signal be given by

$$Y_p(t) = AX_p(t - \Gamma) + N_p(t) \quad (8)$$

where $A > 0$ models the magnitude of the random channel gain/loss and Γ models the delay of the single path from the transmitter to the receiver. $N_p(t)$ is the AWGN.

The receiver is equipped with a local oscillator whose phase at time t is $2\pi f_c t + \phi$. Further the receiver does not know the exact value of A, Γ, θ (The receiver of course has perfect knowledge of its own LO's phase offset ϕ).

Assume the channel parameters, i.e., A, Γ, θ change so rapidly that estimation of these unknown parameters using training is not feasible.

Formulate a generalized likelihood detector for detecting the transmitted information signal non-coherently (i.e., without explicitly estimating the channel parameters).

Given a received passband waveform $y_p(t)$, show that the generalized likelihood detector is given by

$$\left(\hat{X}(t), \hat{\Gamma}\right) = \arg \max_{x(t) \in \mathcal{X}, \tau \geq 0} \frac{\left| \int_{\tau}^{\tau+T_0} y_p(t) e^{j(2\pi f_c t + \phi)} x(t - \tau) dt \right|^2}{\int_0^{T_0} |x(t)|^2 dt} - \frac{1}{2} \int_{\tau}^{\tau+T_0} y_p^2(t) dt \quad (9)$$

where \mathcal{X} is the set of all complex baseband information bearing transmit signals.

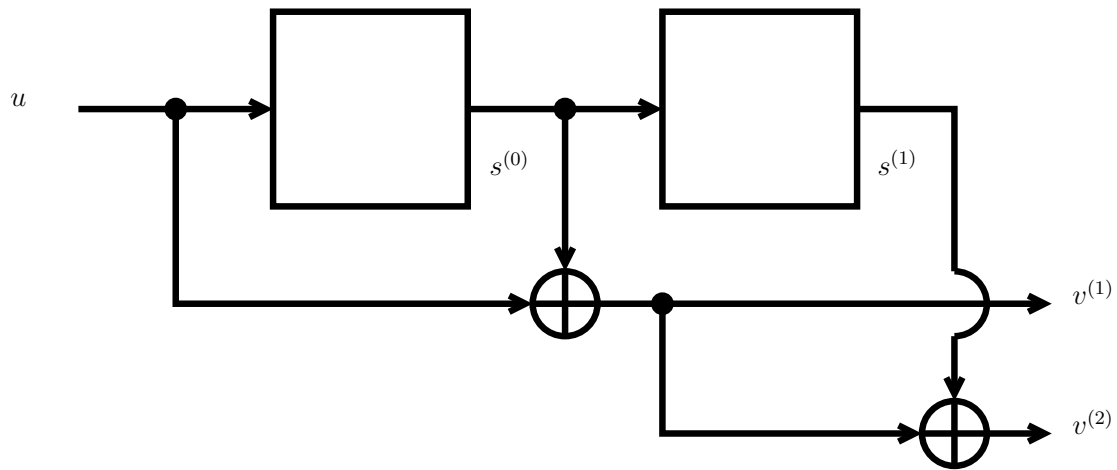


Fig. 1. Rate-1/2 binary convolutional encoder.

5) (15 points) Consider a rate $R = 1/2$ binary convolutional encoder with generator matrix

$$G(D) = \begin{bmatrix} 1 + D & 1 + D + D^2 \end{bmatrix} \quad (10)$$

as shown in Fig. 1.

Draw state diagram and trellis for the binary convolutional encoder with generator matrix given in (10).