Exam in TSKS04 Digital Communication Continuation Course

Exam Code:	TEN1
Date & Time:	8:00 - 12:00, 17 August, 2012
Place:	TER4
Teacher:	Saif Khan Mohammed, tel: 281386
Exam Visit:	08:45 and 11:00
Department:	ISY
Allowed aids:	Olofsson: Tables and Formulas for Signal Theory
	U. Madhow: Fundamentals of Digital Communication
	Calculator with clear memory
Grade Translation:	Grades 3,4, and 5, are translated to ECTS C,B, and A
Solutions:	Within four working days after the exam on the course web page
Grading:	Maximum points: 100, Pass: > 41,
	Grade 3: $42 - 59$, Grade 4: $60 - 79$, Grade 5: $80 - 100$
Important Instructions:	All answers must be given in English
	Please write legibly since partial points will be awarded for each question even if the final answer is incorrect

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1) **Linearly Modulated Signals: Inducing wide sense stationarity using random delay** (30 points)

Consider the linearly modulated signal

$$s(t) = \sum_{n=-\infty}^{\infty} b[n]p(t - nT). \quad (1)$$

a) (4 points) **Show that s is cyclostationary with respect to the interval T if $\{b[n]\}$ is a stationary discrete-time sequence.**

b) (6 points) **Show that s is wide sense cyclostationary with respect to the interval T if $\{b[n]\}$ is a wide sense stationary (WSS) sequence.**

c) (20 points) Assume that $\{b[n]\}$ is a zero mean wide sense stationary (WSS) sequence with autocorrelation function $R_b[k] = \mathbb{E}\big[b[n]b^*[n-k]\big]$. Let $v(t) = s(t - D)$, where D is a random variable distributed uniformly in $[0\ ,\ T]$ (D is independent of s).

Show that v is wide sense stationary.
(Hint: Show that the mean of $v(t)$ is independent of t. Compute the autocorrelation function of v (i.e., $R_v(t_1, t_2) = \mathbb{E}[v(t_1)v^*(t_2)]$) and show that it depends on t_1 and t_2 only through the time difference $t_1 - t_2$. Be careful to take the expectation over D also, in addition to taking it over $\{b[n]\}$.)

2) **FSK tone spacing** (20 points)

Consider two real-valued passband pulses of the form

$$\begin{aligned} s_0(t) &= \cos(2\pi f_0 t + \phi_0)\,,\ 0 \le t \le T\,, \\ s_1(t) &= \cos(2\pi f_1 t + \phi_1)\,,\ 0 \le t \le T\,, \end{aligned} \tag{2}$$

where $f_1 > f_0 \gg 1/T$ and $2f_0T\,, 2f_1T$ are integers. The pulses are said to be orthogonal if $\int_0^T s_0(t)s_1(t)\,dt = 0$.

a) (10 points) **If $\phi_0 = \phi_1 = 0$, show that the minimum frequency separation such that the pulses are orthogonal is $f_1 - f_0 = \frac{1}{2T}$.**

b) (10 points) **If ϕ_0 and ϕ_1 are arbitrary phases, show that the minimum separation for the pulses to be orthogonal regardless of ϕ_0, ϕ_1 is $f_1 - f_0 = \frac{1}{T}$.**

(Remark: This is the reason why noncoherent FSK requires twice the bandwidth as that required with coherent FSK.)

3) Maximum Likelihood Sequence Estimation (MLSE) (25 points)

Consider a received signal of the form $y(t) = \sum_l b[l]p(t - lT) + n(t)$, where $b[l] \in \{-1, +1\}$ is the transmitted sequence of information bits, $n(t)$ is AWGN (real), and the pulse shaping waveform $p(t)$ is a real-valued and even function of t (i.e., $p(t) = p^*(t)$, $p(t) = p(-t)$).

At the receiver, the signal $y(t)$ is passed through a receive filter having impulse response $g(t)$ (not necessarily matched to $p(t)$). Suppose that $g(t)$ is such that it satisfies $g(t) \star h(t) = p(t)$ where $h(t) = \delta(t - \frac{T}{4}) + \delta(t + \frac{T}{6})$ (here $\star$ denotes the convolution operation, and $\delta(t)$ denotes the dirac-delta function).

Let $r(t) = y(t) \star g(t)$ denote the output of the receive filter. Let the filtered received signal $r(t)$ be sampled at a rate $1/T_s$ Hz and with a delay of τ seconds, to result in the discrete-time received sequence $r[l] = r(lT_s - \tau)$.

Show that it is possible to implement the MLSE for the original continuous-time signal $y(t)$, using only the discrete-time samples $\{r[l]\}$ for some choice of T_s and τ. Specify a choice of T_s and τ that makes this possible.

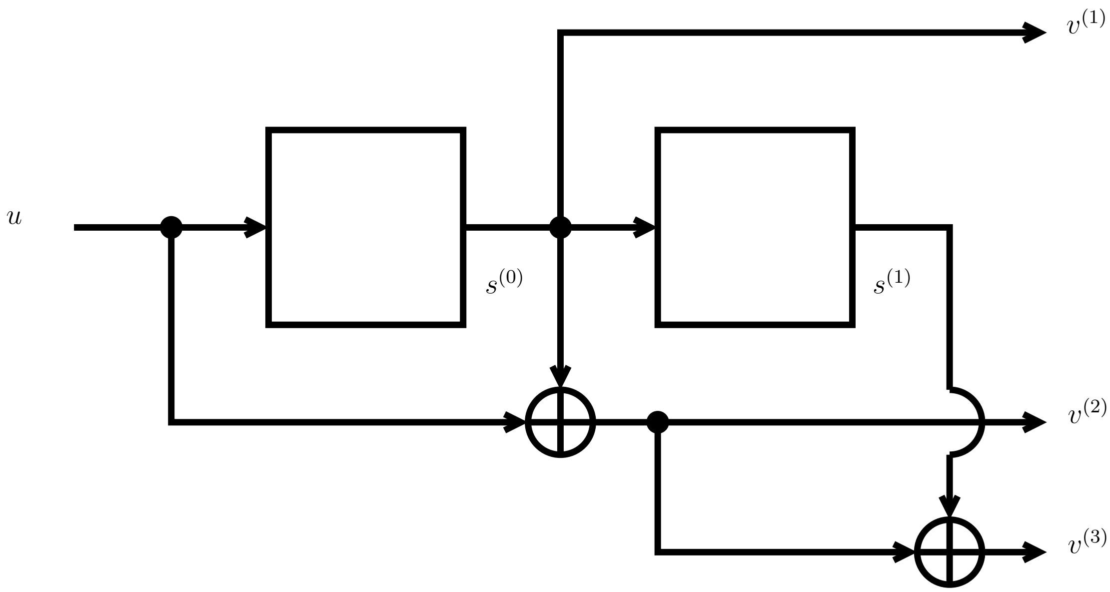

Fig. 1. Rate-1/3 binary convolutional encoder.

4) (25 points) Consider a rate $R = 1/3$ binary convolutional encoder with generator matrix

$$G(D) = \begin{bmatrix} D & 1+D & 1+D+D^2 \end{bmatrix} \tag{3}$$

as shown in Fig. 1.

a) (10 points) **Draw the state diagram for the binary convolutional encoder with generator matrix given in (3).**

b) (15 points) **Draw the signal flow-chart for the convolutional encoder above and compute an expression for the path weight enumerator $T(X) = \sum_i A(i)X^i$ where $A(i)$ is the number of simple-paths (first error event paths in the trellis) whose output sequence has Hamming weight equal to i. Using the expression for $T(X)$, find out the free-distance d_{free} of the above convolutional code, and also, how many different simple-paths exist with Hamming weight equal to 10 ?**