TSKS02 Telecommunication

Solutions for the exam 2017-01-12

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Question part

The questions in this part can give you at most 5 points each. You need at least 5 points from this part of the exam to pass.

1

The purpose of a phase-locked loop is syncronization, to create a local carrier (in a receiver) that is syncronized to the carrier in a received, noisy and modulated signal. A VCO (Voltage Controlled Oscillator) creates a carrier, that is compared to a received signal by multiplying them. The average of the resulting signal is easily related to the difference of the phases. Therefore, the signal is LP-filtered, and the output of that filter controls the frequency of the VCO. The result is a signal that has the correct frequency, and a phase that is easily related to the phase of the received carrier.

2

See Chapter 4.3.2 in 'Telecommunication Methods' by Mikael Olofsson.

3

You do not need to explain your answers in this task. All that is needed is a true or a false. However, a short explanation or comment is given here for some claims.

a. **False**. The speed of light in all materials is less than the speed of light in vacuum.

b. **True**.

c. **False**. It is the other way around. Quantization adds distorsion.

d. **False**. FM is an analog modulation technique.

e. **True**.

Problem part

The problems in this part can give you at most 5 points each. You need at least 5 points from this part of the exam to pass.

4

a. We have the modulated signal

$$x(t) = A \cdot m(t) \cdot \cos(2\pi f_c t).$$

The receiver samples it and forms the time-discrete signal

$$\tilde{x}[n] = x(nT) = A \cdot m(nT) \cdot \cos(2\pi f_c nT),$$

where T is the sampling period. We are given the sampling frequency är f_c, which gives us $T = 1/f_c$. Inserting that in the expression above, we get

$$\tilde{x}[n] = A \cdot m(nT) \cdot \cos(n \cdot 2\pi) = A \cdot m(nT)$$

If we instead had sampled the message $m(t)$, we would have gotten

$$\tilde{m}[n] = m(nT)$$

So, $\tilde{x}[n]$ is an amplitude-scaled version of $\tilde{m}[n]$. The sampling frequency is significantly larger that the bandwidth of the signal, which means that the ideal reconstruction of $\tilde{m}[n]$ does not cause any aliasing. So, we get $m(t)$ from $\tilde{m}[n]$, ocand from $\tilde{x}[n]$ we get $A \cdot m(t)$, i.e. an amplitude-scaled version of the message.

b. This method demands good synchronization. If the sampling instances are shifted due to that the receiver is not synchronized with the sender, and if this shift is $\tau = T\alpha/2\pi$, where T is the period of the carrier, then the demodulated signal is multiplied by $\cos(\alpha)$ compared to the ideal sampling instances. The worst case is if we have $\alpha = \left(k + \frac{1}{2}\right)\pi$, sinc then we have $\cos(\alpha) = 0$. Then the output of the receiver will be zero.

Answer: See above.

5
Since the code is supposed to have dimension $k = 2$ and length $n = 3t + 2$, we need two linearly independent codewords of length $3t + 2$ to form a generator matrix. Those two codewords must have weights at least $2t + 1$, and the distance between them must be at least $2t + 1$. A suitable choice of generator matrix can then be

$$G = \begin{pmatrix} \overbrace{1\dots1}^{t+1} & \overbrace{1\dots1}^{t} & \overbrace{0\dots0}^{t+1} \\ 0\dots0 & 1\dots1 & 1\dots1 \end{pmatrix}.$$

This code consists of the following codewords.

$$\overbrace{0\dots\dots\dots\dots\dots0}^{3t+2},$$
$$\overbrace{1\dots\dots\dots1}^{2t+1}\,\overbrace{0\dots0}^{t+1},$$
$$\overbrace{0\dots0}^{t+1}\,\overbrace{1\dots\dots\dots1}^{2t+1},$$
$$\overbrace{1\dots1}^{t+1}\,\overbrace{0\dots0}^{t}\,\overbrace{1\dots1}^{t+1}.$$

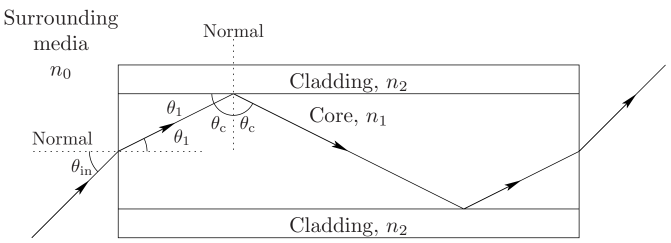
The weights of those codewords are in order

$$0, \qquad 2t + 1, \qquad 2t + 1, \qquad 2t + 2.$$

The smallest non-zero weight is $2t + 1$, as requested. The code has the wanted parameters, and thus such a code exists.

Answer: See above.

6
We are considering a step-index fiber, for which the core has refraction index $n_1 = 1.5$. The numeric aperture is $\sin(\theta_{\text{in}}) = 0.3$ when the surrounding media is air, for which we use the approximative refraction index $n_0 \approx 1$. See the figure below for details.



a. Snells law gives us

$$n_0 \sin(\theta_{\text{in}}) = n_1 \sin(\theta_1),$$

which we rewrite as

$$\sin(\theta_1) = \frac{n_0}{n_1} \sin(\theta_{\text{in}}) = 0.2,$$

where we have plugged in the given numbers. Plain geometry gives us

$$\cos(\theta_{\text{c}}) = \sin(\theta_1) = 0.2,$$

and

$$\sin(\theta_{\text{c}}) = \sqrt{1 - \cos^2(\theta_{\text{c}})} = \sqrt{0.96},$$

where we have plugged in $\cos(\theta_{\text{c}}) = 0.2$ from above. Snells law again, gives us

$$n_1 \sin(\theta_{\text{c}}) = n_2 \sin(\pi/2),$$

which we rewrite as

$$n_2 = \frac{n_1 \sin(\theta_{\text{c}})}{\sin(\pi/2)} = 1.5 \cdot \sqrt{0.96} \approx 0.47,$$

b. The acceptance half-angle is θ_{in}, for which we have according to the problem formulation

$$\sin(\theta_{\text{in}}) = 0.3,$$

which we rewrite as

$$\theta_{\text{in}} = \arcsin(0.3) \approx 0.305 \text{ rad} \approx 17.5°.$$

c. Again we use Snells law,

$$n_0 \sin(\theta_{\text{in}}) = n_1 \sin(\theta_1),$$

but this time with the given $n_0 = 4/3$. We rewrite that to get the numeric aperture as

$$\sin(\theta_{\text{in}}) = \frac{n_1}{n_0} \sin(\theta_1) = \frac{9}{40} = 0.225.$$

The task was a bit unclear about which acceptance angle we meant, full or half. This solution assumes full-angle. The acceptance full-angle is $2\theta_{\text{in}}$, which is given by

$$2\theta_{\text{in}} = 2 \arcsin(0.225) \approx 0.454 \text{ rad} \approx 26.0°.$$

Answer:
a. The cladding has refraction index $n_2 = 1.47$.
b. The acceptance half-angle is 17.5°.
c. Acceptance full-angle: 26.0°
Numeric aperture: 0.225