Exam in
TSKS02 Telecommunication, TEN1

Department of Electrical Engineering (ISY), Linköping University

Exam date: April 9, 2015, 14:00–18:00

Examiner: Emil Björnson, telephone: 286732. The examiner will visit the room twice during the exam: around 15:00 and around 17:00.

Administrator: Carina Lindström, carli78@isy.liu.se, telephone: 284423.

Material: Calculator with empty memory. Language dictionary to/from English.

Grading: This exam consists of two parts, a question part and a problem part. Each question or problem can at most give you 5 points. For passing the exam, you need

- at least 5 points from the question part,
- at least 5 points from the problem part,
- and totally at least 12 points.

Grade limits:

- Grade three: 12 points;
- Grade four: 17 points;
- Grade five: 22 points.

Sloppy solutions and solutions that are hard to read/understand are subject to hard judgement, as are clearly unreasonable answers. Grades 3, 4, and 5 are translated to ECTS C, B, and A. **Important: All solutions must be given in English.**

Exam returns: The exams are returned 2015-04-24 at 12:40-13:00 in Filtret, Building B, ground floor, corridor D, close to Entrance 29. After that occasion, the exams can be picked up at the Student's Office of the Department of Electrical Engineering (ISY), Building B, Corridor D between Entrances B27 and B29.

Solutions: The exam solutions will be available on the TSKS02 exam web page (www.commsys.isy.liu.se/en/student/kurser/tentor?TSKS02) within seven working days after the exam.

Result announcement: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

PLEASE NOTE THAT THE PROBLEMS ARE NOT IN ORDER OF DIFFICULTY. IT IS OFTEN POSSIBLE TO CONTINUE SOLVING (b), EVEN IF ONE DOES NOT FIND AN ANSWER TO (a), ETC.
Good luck!

1 Question part

1. Are the following claims true or false? Read the statements carefully do avoid misunderstandings. You do not need to explain your answers.

   a) The Hamming weight of a binary vector is the number of ones in the vector.

   b) Radio waves travel at the speed of sound.

   c) A sampled signal can not be fully recovered again.

   d) The expected codeword length in source coding is lower bounded by the entropy of the source.

   e) AM is used in the course as an abbreviation for angle modulation.

   For each of the claims above, a correct answer gives you $+1$ point, while an incorrect answer gives you -1 point. No answer gives you 0 points for that claim. The total number of points from these questions cannot be lower than 0.

   (5 p)

2. Describe the Huffmann algorithm. Your aim should be that a reader can implement it based on your description. Provide a simple example for a memoryless source with three outputs (each with non-zero probability).

   (5 p)

3. Describe 8-PSK signaling. In particular, you should do/answer the following:

   a) Draw a signal space diagram.

   b) Give a mathematical expression for the signal points $s_i(t)$ for $i = 1, 2, \dots, 8$.

   c) If E_{avg} is the average signal energy, how does it vary of the signal points?

   d) Assign bits to the signals points by following th Gray coding principle.

   (5 p)

2 Problem part

1. Consider a step-index fiber, for which the core has refraction index 1.6. Suppose that $\theta_{in} = \pi/4$ is the acceptance half-angle of the fiber for a particular surrounding media.

   a) Compute the refraction index of the cladding if the surrounding media is air ($n = 1$).

   b) Compute the refraction index of the cladding if the surrounding media is water ($n = 1.3$).

   (5 p)

2. Consider an error-correcting code of length n with the two codewords $c^{(0)} = (0, 0, \ldots, 0)$ and $c^{(1)} = (1, 1, \ldots, 1)$, where n is odd. Answer the following:

   a) Give a generator matrix and a parity check matrix for this code.

   b) How many errors can be detected?

   c) How many errors can be corrected?

   d) Show that the code satisfies the Hamming bound with equality:

   $$2^n = 2^k \sum_{i=0}^{\lfloor \frac{d-1}{2} \rfloor} \binom{n}{i}$$

   (i.e., it is a *perfect* code).

   (5 p)

3. Consider the signal $m(t) = 5\sin(200\pi t)$ which is frequency modulated as $x(t) = \cos(2 \cdot 10^6 \pi t + \int m(t)dt)$.

   a) Determine the frequency deviation.

   b) Determine the peak frequency deviation.

   c) Determine the frequency modulation index.

   (5 p)

Equation Service

Below, E denotes *average* signal energy.

Snell's law:	$n_1 \sin\theta_1 = n_2 \sin\theta_2$
Convolution:	$(a * b)(t) \triangleq \int\limits_{-\infty}^{\infty} a(\tau)b(t-\tau)d\tau$
Fourier transform:	$\mathcal{F}\left\{x(t)\right\} \triangleq \int\limits_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$
Inverse Fourier transform:	$\mathcal{F}^{-1}\left\{X(f)\right\} \triangleq \int\limits_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$
Parseval's relation:	$\int\limits_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \int\limits_{-\infty}^{\infty} \lvert X(f)\rvert^2 df$
Derivatives:	$\mathcal{F}\left\{\frac{d^n}{dt^n}x(t)\right\} = (j2\pi f)^n X(f)$
Stationary cosine:	$\mathcal{F}\left\{\cos\left(2\pi f_{\mathrm{c}} t\right)\right\} = \frac{1}{2}\left(\delta(f+f_{\mathrm{c}}) + \delta(f-f_{\mathrm{c}})\right)$
Stationary sine:	$\mathcal{F}\left\{\sin\left(2\pi f_{\mathrm{c}} t\right)\right\} = \frac{j}{2}\left(\delta(f+f_{\mathrm{c}}) - \delta(f-f_{\mathrm{c}})\right)$
Time-discrete Fourier transform:	$\mathcal{F}\left\{x[n]\right\} \triangleq \sum\limits_{n=-\infty}^{\infty} x[n]e^{-j2\pi\theta n}$
Time-discrete Inverse Fourier transform:	$\mathcal{F}^{-1}\left\{X[\theta]\right\} \triangleq \int\limits_{-1/2}^{1/2} X[\theta]e^{j2\pi\theta n}d\theta$
Poisson's summation formula:	$\tilde{X}[\theta] = \frac{1}{T}\sum\limits_{k=-\infty}^{\infty} X\left(\frac{\theta-k}{T}\right)$
Pulse Amplitude Modulation:	$Y(f) = P(f)\tilde{X}[fT]$
OOK & BFSK:	$P_{\mathrm{e}} = Q\left(\sqrt{\frac{E}{N_0}}\right)$
BPSK:	$P_{\mathrm{e}} = Q\left(\sqrt{\frac{2E}{N_0}}\right)$
4-ASK:	$P_{\mathrm{e}} = \frac{3}{2}Q\left(\sqrt{\frac{2E}{5N_0}}\right)$
M-PSK ($M > 2$):	$P_{\mathrm{e}} \approx 2Q\left(\sqrt{\frac{2E}{N_0}}\sin(\frac{\pi}{M})\right)$
16-QAM:	$P_{\mathrm{e}} \approx 3Q\left(\sqrt{\frac{E}{5N_0}}\right)$
M-FSK:	$P_{\mathrm{e}} \approx (M-1)Q\left(\sqrt{\frac{E}{N_0}}\right)$
Std integral:	$\int \frac{1}{1+x^2}dx = \arctan(x) + C$
Kraft's inequality:	$\sum\limits_i 2^{-l_i} \leq 1$

The Q-function, table of $Q(x) = \int\limits_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ for $0.00 \leq x \leq 5.99$.

x	0	1	2	3	4	5	6	7	8	9	exp
0.0	5.0000	4.9601	4.9202	4.8803	4.8405	4.8006	4.7608	4.7210	4.6812	4.6414	
0.1	4.6017	4.5620	4.5224	4.4828	4.4433	4.4038	4.3644	4.3251	4.2858	4.2465	
0.2	4.2074	4.1683	4.1294	4.0905	4.0517	4.0129	3.9743	3.9358	3.8974	3.8591	
0.3	3.8209	3.7828	3.7448	3.7070	3.6693	3.6317	3.5942	3.5569	3.5197	3.4827	
0.4	3.4458	3.4090	3.3724	3.3360	3.2997	3.2636	3.2276	3.1918	3.1561	3.1207	
0.5	3.0854	3.0503	3.0153	2.9806	2.9460	2.9116	2.8774	2.8434	2.8096	2.7760	
0.6	2.7425	2.7093	2.6763	2.6435	2.6109	2.5785	2.5463	2.5143	2.4825	2.4510	−1
0.7	2.4196	2.3885	2.3576	2.3270	2.2965	2.2663	2.2363	2.2065	2.1770	2.1476	
0.8	2.1186	2.0897	2.0611	2.0327	2.0045	1.9766	1.9489	1.9215	1.8943	1.8673	
0.9	1.8406	1.8141	1.7879	1.7619	1.7361	1.7106	1.6853	1.6602	1.6354	1.6109	
1.0	1.5866	1.5625	1.5386	1.5151	1.4917	1.4686	1.4457	1.4231	1.4007	1.3786	
1.1	1.3567	1.3350	1.3136	1.2924	1.2714	1.2507	1.2302	1.2100	1.1900	1.1702	
1.2	1.1507	1.1314	1.1123	1.0935	1.0749	1.0565	1.0383	1.0204	1.0027	9.8525	
1.3	9.6800	9.5098	9.3418	9.1759	9.0123	8.8508	8.6915	8.5343	8.3793	8.2264	
1.4	8.0757	7.9270	7.7804	7.6359	7.4934	7.3529	7.2145	7.0781	6.9437	6.8112	
1.5	6.6807	6.5522	6.4255	6.3008	6.1780	6.0571	5.9380	5.8208	5.7053	5.5917	
1.6	5.4799	5.3699	5.2616	5.1551	5.0503	4.9471	4.8457	4.7460	4.6479	4.5514	
1.7	4.4565	4.3633	4.2716	4.1815	4.0930	4.0059	3.9204	3.8364	3.7538	3.6727	−2
1.8	3.5930	3.5148	3.4380	3.3625	3.2884	3.2157	3.1443	3.0742	3.0054	2.9379	
1.9	2.8717	2.8067	2.7429	2.6803	2.6190	2.5588	2.4998	2.4419	2.3852	2.3295	
2.0	2.2750	2.2216	2.1692	2.1178	2.0675	2.0182	1.9699	1.9226	1.8763	1.8309	
2.1	1.7864	1.7429	1.7003	1.6586	1.6177	1.5778	1.5386	1.5003	1.4629	1.4262	
2.2	1.3903	1.3553	1.3209	1.2874	1.2545	1.2224	1.1911	1.1604	1.1304	1.1011	
2.3	1.0724	1.0444	1.0170	9.9031	9.6419	9.3867	9.1375	8.8940	8.6563	8.4242	
2.4	8.1975	7.9763	7.7603	7.5494	7.3436	7.1428	6.9469	6.7557	6.5691	6.3872	
2.5	6.2097	6.0366	5.8677	5.7031	5.5426	5.3861	5.2336	5.0849	4.9400	4.7988	
2.6	4.6612	4.5271	4.3965	4.2692	4.1453	4.0246	3.9070	3.7926	3.6811	3.5726	
2.7	3.4670	3.3642	3.2641	3.1667	3.0720	2.9798	2.8901	2.8028	2.7179	2.6354	−3
2.8	2.5551	2.4771	2.4012	2.3274	2.2557	2.1860	2.1182	2.0524	1.9884	1.9262	
2.9	1.8658	1.8071	1.7502	1.6948	1.6411	1.5889	1.5382	1.4890	1.4412	1.3949	
3.0	1.3499	1.3062	1.2639	1.2228	1.1829	1.1442	1.1067	1.0703	1.0350	1.0008	
3.1	9.6760	9.3544	9.0426	8.7403	8.4474	8.1635	7.8885	7.6219	7.3638	7.1136	
3.2	6.8714	6.6367	6.4095	6.1895	5.9765	5.7703	5.5706	5.3774	5.1904	5.0094	
3.3	4.8342	4.6648	4.5009	4.3423	4.1889	4.0406	3.8971	3.7584	3.6243	3.4946	
3.4	3.3693	3.2481	3.1311	3.0179	2.9086	2.8029	2.7009	2.6023	2.5071	2.4151	−4
3.5	2.3263	2.2405	2.1577	2.0778	2.0006	1.9262	1.8543	1.7849	1.7180	1.6534	
3.6	1.5911	1.5310	1.4730	1.4171	1.3632	1.3112	1.2611	1.2128	1.1662	1.1213	
3.7	1.0780	1.0363	9.9611	9.5740	9.2010	8.8417	8.4957	8.1624	7.8414	7.5324	
3.8	7.2348	6.9483	6.6726	6.4072	6.1517	5.9059	5.6694	5.4418	5.2228	5.0122	
3.9	4.8096	4.6148	4.4274	4.2473	4.0741	3.9076	3.7475	3.5936	3.4458	3.3037	−5
4.0	3.1671	3.0359	2.9099	2.7888	2.6726	2.5609	2.4536	2.3507	2.2518	2.1569	
4.1	2.0658	1.9783	1.8944	1.8138	1.7365	1.6624	1.5912	1.5230	1.4575	1.3948	
4.2	1.3346	1.2769	1.2215	1.1685	1.1176	1.0689	1.0221	9.7736	9.3447	8.9337	
4.3	8.5399	8.1627	7.8015	7.4555	7.1241	6.8069	6.5031	6.2123	5.9340	5.6675	
4.4	5.4125	5.1685	4.9350	4.7117	4.4979	4.2935	4.0980	3.9110	3.7322	3.5612	−6
4.5	3.3977	3.2414	3.0920	2.9492	2.8127	2.6823	2.5577	2.4386	2.3249	2.2162	
4.6	2.1125	2.0133	1.9187	1.8283	1.7420	1.6597	1.5810	1.5060	1.4344	1.3660	
4.7	1.3008	1.2386	1.1792	1.1226	1.0686	1.0171	9.6796	9.2113	8.7648	8.3391	
4.8	7.9333	7.5465	7.1779	6.8267	6.4920	6.1731	5.8693	5.5799	5.3043	5.0418	
4.9	4.7918	4.5538	4.3272	4.1115	3.9061	3.7107	3.5247	3.3476	3.1792	3.0190	−7
5.0	2.8665	2.7215	2.5836	2.4524	2.3277	2.2091	2.0963	1.9891	1.8872	1.7903	
5.1	1.6983	1.6108	1.5277	1.4487	1.3737	1.3024	1.2347	1.1705	1.1094	1.0515	
5.2	9.9644	9.4420	8.9462	8.4755	8.0288	7.6050	7.2028	6.8212	6.4592	6.1158	
5.3	5.7901	5.4813	5.1884	4.9106	4.6473	4.3977	4.1611	3.9368	3.7243	3.5229	
5.4	3.3320	3.1512	2.9800	2.8177	2.6640	2.5185	2.3807	2.2502	2.1266	2.0097	−8
5.5	1.8990	1.7942	1.6950	1.6012	1.5124	1.4283	1.3489	1.2737	1.2026	1.1353	
5.6	1.0718	1.0116	9.5479	9.0105	8.5025	8.0224	7.5686	7.1399	6.7347	6.3520	
5.7	5.9904	5.6488	5.3262	5.0215	4.7338	4.4622	4.2057	3.9636	3.7350	3.5193	
5.8	3.3157	3.1236	2.9424	2.7714	2.6100	2.4579	2.3143	2.1790	2.0513	1.9310	−9
5.9	1.8175	1.7105	1.6097	1.5147	1.4251	1.3407	1.2612	1.1863	1.1157	1.0492	

For $x > 0$, we have $(1 - x^{-2})\frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt < Q(x) < \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$. For large x we have $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$.