Exam in
TSKS02 Telecommunication, TEN1

Department of Electrical Engineering (ISY), Linköping University

Exam date: January 15, 2015, 14:00–18:00

Examiner: Emil Björnson, telephone: 286732. The examiner will visit the room twice during the exam: around 15:00 and around 17:00.

Administrator: Carina Lindström, carli78@isy.liu.se, telephone: 284423.

Material: Calculator with empty memory. Language dictionary to/from English.

Grading: This exam consists of two parts, a question part and a problem part. Each question or problem can at most give you 5 points. For passing the exam, you need

- at least 5 points from the question part,
- at least 5 points from the problem part,
- and totally at least 12 points.

Grade limits:

- Grade three: 12 points;
- Grade four: 17 points;
- Grade five: 22 points.

Sloppy solutions and solutions that are hard to read/understand are subject to hard judgement, as are clearly unreasonable answers. Grades 3, 4, and 5 are translated to ECTS C, B, and A. **Important: All solutions must be given in English.**

Exam returns: The exams are returned 2015-01-28 at 12:40-13:00 in Emil's office, Building B, Ground floor, Corridor D between Entrances 25 and 27. After that occasion, the exams can be picked up at the Student's Office of the Dept. of EE. (ISY), Building B, Corridor D between Entrances B27 and B29.

Solutions: The exam solutions will be available on the TSKS02 exam web page (www.commsys.isy.liu.se/en/student/kurser/tentor?TSKS02) within five working days after the exam.

Result announcement: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

PLEASE NOTE THAT THE PROBLEMS ARE NOT IN ORDER OF DIFFICULTY. IT IS OFTEN POSSIBLE TO CONTINUE SOLVING (b), EVEN IF ONE DOES NOT FIND AN ANSWER TO (a), ETC.
Good luck!

1 Question part

1. Are the following claims true or false? You do not need to explain your answers.

   a) Optical fibers have large refraction in the core and smaller in the cladding.

   b) Aliasing can appear when the carrier in AM is suppressed.

   c) The bit error probability of 4-PSK can be reduced by error control coding.

   d) The cable length determines how strong the fading phenomenon is.

   e) The Hamming bound gives the shortest codeword length in source coding.

   For each of the claims above, a correct answer gives you $+1$ point, while an incorrect answer gives you -1 point. No answer gives you 0 points for that claim. The total number of points from these questions cannot be lower than 0.

   (5 p)

2. Let $x(t)$ be a signal that is zero outside $t \in [0, T)$. Assume that $x(t)$ can be described by two basis functions.

   a) Define a vector representation of $x(t)$.

   b) Define and explain the concept of *orthogonal* basis functions.

   c) Explain why it is convenient to normalize basis functions to have energy 1.

   d) Give an example of two basis functions that are orthogonal and normalized.

   e) Explain what a *signal constellation* is.

   (5 p)

3. Describe the generator matrix of a linear code. What does it mean to write it on systematic form?
   If we have a non-systematic generator matrix, how can we rewrite it on systematic form? Give a concrete example.

   (5 p)

2 Problem part

1. Consider an LTE system characterized by the following differential equation:

$$\frac{d^2}{dt^2}y(t) - 4y(t) = 3\frac{d}{dt}x(t)$$

where $x(t)$ is the input signal and $y(t)$ is the output signal.

a) Determine the frequency response of the system.

b) Determine the amplitude characteristic of the system.

c) Determine the phase characteristic of the system.

d) Determine the output signal $y(t)$ corresponding to the input signal $x(t) = \sin\left(2\pi t + \frac{\pi}{4}\right)$.

(5 p)

2. Suppose that a digital communication system requires that the bit error probability, P_b, is below 10^{-4}. You can choose between the following two modulations:

- BPSK
- 4-PSK

Determine the smallest possible signal-to-noise ratio (SNR) that gives $P_b \lesssim 10^{-4}$ for each of the modulations.
State which one that is the better choice. Give a short explanation of why this modulation can handle lower SNRs than the other one.

(5 p)

3. Suppose that each voter in an election selects a party in the following way:

Party	Probability
A	36 %
B	28 %
C	18 %
D	11 %
E	7 %

Let the party selection be the source of information that shall be sent over a channel.

a) Construct a code for this source by using the *full* Huffman algorithm (not the simplified one).

b) Draw the tree that describes your code.

c) Determine the average codeword length of the code.

d) Determine the redundancy and the compression ratio of the code.

(5 p)

Equation Service

Below, E denotes *average* signal energy.

Snell's law:	$n_1 \sin\theta_1 = n_2 \sin\theta_2$
Convolution:	$(a * b)(t) \triangleq \int\limits_{-\infty}^{\infty} a(\tau)b(t-\tau)d\tau$
Fourier transform:	$\mathcal{F}\{x(t)\} \triangleq \int\limits_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$
Inverse Fourier transform:	$\mathcal{F}^{-1}\{X(f)\} \triangleq \int\limits_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$
Parseval's relation:	$\int\limits_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \int\limits_{-\infty}^{\infty} \lvert X(f)\rvert^2 df$
Derivatives:	$\mathcal{F}\left\{\frac{d^n}{dt^n}x(t)\right\} = (j2\pi f)^n X(f)$
Stationary cosine:	$\mathcal{F}\{\cos(2\pi f_c t)\} = \frac{1}{2}\left(\delta(f+f_c)+\delta(f-f_c)\right)$
Stationary sine:	$\mathcal{F}\{\sin(2\pi f_c t)\} = \frac{j}{2}\left(\delta(f+f_c)-\delta(f-f_c)\right)$
Time-discrete Fourier transform:	$\mathcal{F}\{x[n]\} \triangleq \sum\limits_{n=-\infty}^{\infty} x[n]e^{-j2\pi\theta n}$
Time-discrete Inverse Fourier transform:	$\mathcal{F}^{-1}\{X[\theta]\} \triangleq \int\limits_{-1/2}^{1/2} X[\theta]e^{j2\pi\theta n}d\theta$
Poisson's summation formula:	$\tilde{X}[\theta] = \frac{1}{T}\sum\limits_{k=-\infty}^{\infty} X\left(\frac{\theta-k}{T}\right)$
Pulse Amplitude Modulation:	$Y(f) = P(f)\tilde{X}[fT]$
OOK & BFSK:	$P_e = Q\left(\sqrt{\frac{E}{N_0}}\right)$
BPSK:	$P_e = Q\left(\sqrt{\frac{2E}{N_0}}\right)$
4-ASK:	$P_e = \frac{3}{2}Q\left(\sqrt{\frac{2E}{5N_0}}\right)$
M-PSK ($M > 2$):	$P_e \approx 2Q\left(\sqrt{\frac{2E}{N_0}}\sin(\frac{\pi}{M})\right)$
16-QAM:	$P_e \approx 3Q\left(\sqrt{\frac{E}{5N_0}}\right)$
M-FSK:	$P_e \approx (M-1)Q\left(\sqrt{\frac{E}{N_0}}\right)$
Std integral:	$\int \frac{1}{1+x^2}dx = \arctan(x) + C$
Kraft's inequality:	$\sum\limits_i 2^{-l_i} \leq 1$

The Q-function, table of $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ for $0.00 \le x \le 5.99$.

x	0	1	2	3	4	5	6	7	8	9	exp
0.0	5.0000	4.9601	4.9202	4.8803	4.8405	4.8006	4.7608	4.7210	4.6812	4.6414	
0.1	4.6017	4.5620	4.5224	4.4828	4.4433	4.4038	4.3644	4.3251	4.2858	4.2465	
0.2	4.2074	4.1683	4.1294	4.0905	4.0517	4.0129	3.9743	3.9358	3.8974	3.8591	
0.3	3.8209	3.7828	3.7448	3.7070	3.6693	3.6317	3.5942	3.5569	3.5197	3.4827	
0.4	3.4458	3.4090	3.3724	3.3360	3.2997	3.2636	3.2276	3.1918	3.1561	3.1207	
0.5	3.0854	3.0503	3.0153	2.9806	2.9460	2.9116	2.8774	2.8434	2.8096	2.7760	
0.6	2.7425	2.7093	2.6763	2.6435	2.6109	2.5785	2.5463	2.5143	2.4825	2.4510	−1
0.7	2.4196	2.3885	2.3576	2.3270	2.2965	2.2663	2.2363	2.2065	2.1770	2.1476	
0.8	2.1186	2.0897	2.0611	2.0327	2.0045	1.9766	1.9489	1.9215	1.8943	1.8673	
0.9	1.8406	1.8141	1.7879	1.7619	1.7361	1.7106	1.6853	1.6602	1.6354	1.6109	
1.0	1.5866	1.5625	1.5386	1.5151	1.4917	1.4686	1.4457	1.4231	1.4007	1.3786	
1.1	1.3567	1.3350	1.3136	1.2924	1.2714	1.2507	1.2302	1.2100	1.1900	1.1702	
1.2	1.1507	1.1314	1.1123	1.0935	1.0749	1.0565	1.0383	1.0204	1.0027	9.8525	
1.3	9.6800	9.5098	9.3418	9.1759	9.0123	8.8508	8.6915	8.5343	8.3793	8.2264	
1.4	8.0757	7.9270	7.7804	7.6359	7.4934	7.3529	7.2145	7.0781	6.9437	6.8112	
1.5	6.6807	6.5522	6.4255	6.3008	6.1780	6.0571	5.9380	5.8208	5.7053	5.5917	
1.6	5.4799	5.3699	5.2616	5.1551	5.0503	4.9471	4.8457	4.7460	4.6479	4.5514	
1.7	4.4565	4.3633	4.2716	4.1815	4.0930	4.0059	3.9204	3.8364	3.7538	3.6727	−2
1.8	3.5930	3.5148	3.4380	3.3625	3.2884	3.2157	3.1443	3.0742	3.0054	2.9379	
1.9	2.8717	2.8067	2.7429	2.6803	2.6190	2.5588	2.4998	2.4419	2.3852	2.3295	
2.0	2.2750	2.2216	2.1692	2.1178	2.0675	2.0182	1.9699	1.9226	1.8763	1.8309	
2.1	1.7864	1.7429	1.7003	1.6586	1.6177	1.5778	1.5386	1.5003	1.4629	1.4262	
2.2	1.3903	1.3553	1.3209	1.2874	1.2545	1.2224	1.1911	1.1604	1.1304	1.1011	
2.3	1.0724	1.0444	1.0170	9.9031	9.6419	9.3867	9.1375	8.8940	8.6563	8.4242	
2.4	8.1975	7.9763	7.7603	7.5494	7.3436	7.1428	6.9469	6.7557	6.5691	6.3872	
2.5	6.2097	6.0366	5.8677	5.7031	5.5426	5.3861	5.2336	5.0849	4.9400	4.7988	
2.6	4.6612	4.5271	4.3965	4.2692	4.1453	4.0246	3.9070	3.7926	3.6811	3.5726	
2.7	3.4670	3.3642	3.2641	3.1667	3.0720	2.9798	2.8901	2.8028	2.7179	2.6354	−3
2.8	2.5551	2.4771	2.4012	2.3274	2.2557	2.1860	2.1182	2.0524	1.9884	1.9262	
2.9	1.8658	1.8071	1.7502	1.6948	1.6411	1.5889	1.5382	1.4890	1.4412	1.3949	
3.0	1.3499	1.3062	1.2639	1.2228	1.1829	1.1442	1.1067	1.0703	1.0350	1.0008	
3.1	9.6760	9.3544	9.0426	8.7403	8.4474	8.1635	7.8885	7.6219	7.3638	7.1136	
3.2	6.8714	6.6367	6.4095	6.1895	5.9765	5.7703	5.5706	5.3774	5.1904	5.0094	
3.3	4.8342	4.6648	4.5009	4.3423	4.1889	4.0406	3.8971	3.7584	3.6243	3.4946	
3.4	3.3693	3.2481	3.1311	3.0179	2.9086	2.8029	2.7009	2.6023	2.5071	2.4151	−4
3.5	2.3263	2.2405	2.1577	2.0778	2.0006	1.9262	1.8543	1.7849	1.7180	1.6534	
3.6	1.5911	1.5310	1.4730	1.4171	1.3632	1.3112	1.2611	1.2128	1.1662	1.1213	
3.7	1.0780	1.0363	9.9611	9.5740	9.2010	8.8417	8.4957	8.1624	7.8414	7.5324	
3.8	7.2348	6.9483	6.6726	6.4072	6.1517	5.9059	5.6694	5.4418	5.2228	5.0122	
3.9	4.8096	4.6148	4.4274	4.2473	4.0741	3.9076	3.7475	3.5936	3.4458	3.3037	−5
4.0	3.1671	3.0359	2.9099	2.7888	2.6726	2.5609	2.4536	2.3507	2.2518	2.1569	
4.1	2.0658	1.9783	1.8944	1.8138	1.7365	1.6624	1.5912	1.5230	1.4575	1.3948	
4.2	1.3346	1.2769	1.2215	1.1685	1.1176	1.0689	1.0221	9.7736	9.3447	8.9337	
4.3	8.5399	8.1627	7.8015	7.4555	7.1241	6.8069	6.5031	6.2123	5.9340	5.6675	
4.4	5.4125	5.1685	4.9350	4.7117	4.4979	4.2935	4.0980	3.9110	3.7322	3.5612	−6
4.5	3.3977	3.2414	3.0920	2.9492	2.8127	2.6823	2.5577	2.4386	2.3249	2.2162	
4.6	2.1125	2.0133	1.9187	1.8283	1.7420	1.6597	1.5810	1.5060	1.4344	1.3660	
4.7	1.3008	1.2386	1.1792	1.1226	1.0686	1.0171	9.6796	9.2113	8.7648	8.3391	
4.8	7.9333	7.5465	7.1779	6.8267	6.4920	6.1731	5.8693	5.5799	5.3043	5.0418	
4.9	4.7918	4.5538	4.3272	4.1115	3.9061	3.7107	3.5247	3.3476	3.1792	3.0190	−7
5.0	2.8665	2.7215	2.5836	2.4524	2.3277	2.2091	2.0963	1.9891	1.8872	1.7903	
5.1	1.6983	1.6108	1.5277	1.4487	1.3737	1.3024	1.2347	1.1705	1.1094	1.0515	
5.2	9.9644	9.4420	8.9462	8.4755	8.0288	7.6050	7.2028	6.8212	6.4592	6.1158	
5.3	5.7901	5.4813	5.1884	4.9106	4.6473	4.3977	4.1611	3.9368	3.7243	3.5229	
5.4	3.3320	3.1512	2.9800	2.8177	2.6640	2.5185	2.3807	2.2502	2.1266	2.0097	−8
5.5	1.8990	1.7942	1.6950	1.6012	1.5124	1.4283	1.3489	1.2737	1.2026	1.1353	
5.6	1.0718	1.0116	9.5479	9.0105	8.5025	8.0224	7.5686	7.1399	6.7347	6.3520	
5.7	5.9904	5.6488	5.3262	5.0215	4.7338	4.4622	4.2057	3.9636	3.7350	3.5193	
5.8	3.3157	3.1236	2.9424	2.7714	2.6100	2.4579	2.3143	2.1790	2.0513	1.9310	−9
5.9	1.8175	1.7105	1.6097	1.5147	1.4251	1.3407	1.2612	1.1863	1.1157	1.0492	

For $x > 0$, we have $(1 - x^{-2})\frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt < Q(x) < \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$. For large x we have $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$.