Exam in
TSKS02 Telecommunication, TEN1

Department of Electrical Engineering (ISY), Linköping University

Exam date: August 30, 2014, 8:00–12:00

Examiner: Emil Björnson, tel.: 281000 (previous examiner Daniel Persson was responsible for exam construction). The examiner will visit the room twice during the exam: around 9:00 and around 11:00.

Administrator: Carina Lindström, carli78@isy.liu.se, tel: 284423.

Material: Calculator with empty memory. Language dictionary to/from English.

Grading: This exam consists of two parts, a question part and a problem part. Each question or problem can at most give you 5 points. For passing the exam, you need

- at least 5 points from the question part,
- at least 5 points from the problem part,
- and totally at least 12 points.

Grade limits:

- Grade three: 12 points;
- Grade four: 17 points;
- Grade five: 22 points.

Sloppy solutions and solutions that are hard to read/understand are subject to hard judgement, as are clearly unreasonable answers. Grades 3, 4, and 5 are translated to ECTS C, B, and A. **Important: All solutions must be given in English.**

Exam returns: The exams are returned 2014-09-15 at 12:40-13:00 in Emil's office, Building B, entrance B29, top floor, corridor A. The exams can also be picked up at the Student's Office of the Dept. of EE. (ISY), Building B, Corridor D between Entrances B27 and B29.

Solutions: The exam solutions will be available on the TSKS02 exam web page (www.commsys.isy.liu.se/en/student/kurser/tentor?TSKS02) within five working days after the exam.

Result announcement: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY IN ORDER OF DIFFICULTY. IT IS OFTEN POSSIBLE TO CONTINUE SOLVING (b), EVEN IF ONE DOES NOT FIND AN ANSWER TO (a) ETC. Good luck!

1 Question part

1. Are the following claims true or false? You do not need to explain your answers.

   a) Fading is caused by multi-path propagation.

   b) The envelope detector can be used for demodulation of AM-SSB.

   c) You need the carrier frequency and the correct phase at the receiver to demodulate standard AM-DSB.

   d) Delta modulation can reduce the number of bits required to describe a slowly varying source in comparison with standard quantization.

   e) Kraft's inequality can be used to explain the Hamming bound.

   For each of the claims above, a correct answer gives you $+1$ point, while an incorrect answer gives you -1 point. No answer gives you 0 points for that claim. The total number of points from these questions cannot be lower than 0.

   (5 p)

2. a) Explain what is the *refraction index* of a material.

   b) State *Snell's law* and explain what it describes using a figure.

   c) Explain the concept of *critical angle* and use Snell's law to derive an expression for it.

   d) Explain what is a *step-index* fiber.

   e) Explain what is the *acceptance half-angle* of the fiber and state the formula for the *numeric aperture*.

   (5 p)

3. Describe QAM-signaling.

   (5 p)

2 Problem part

1. Determine the bit signal-to-noise ratio E_b/N_0 required to achieve a bit-error probability of 10^{-4} when using 4-PSK over an AWGN channel. Gray coding is used for the mapping of the bits to symbols. What is the bit-error probability of BPSK for the same bit signal-to-noise ratio? Compare the two modulation schemes in terms of the computed bit-error probabilities and the rates that are achieved.

(5 p)

2. a) Is the code consisting of only the codewords (11011) and (01101) linear? Motivate your answer!
   b) Write a generator matrix on systematic form for a code containing the two codewords above (and possibly additional codewords).
   c) Is the code generated by the generator matrix linear?
   d) Get a parity check matrix for the code.
   e) Calculate the minimum distance of the code!

(5 p)

3. Consider a sender that, given an input message signal $m(t)$, produces the output

$$y(t) = (m * h_1)(t) \cdot \sin(2\pi f_c t).$$

The filter with impulse response $h_1(t)$ is an ideal low-pass filter with cut-off frequency f_0. A possible receiver starts by producing

$$w(t) = y(t) \cdot \sin(2\pi f_c t + \phi).$$

Its output, $z(t)$, is the given by filtering $w(t)$ using an ideal low-pass filter with impulse response $h_2(t)$. The cut-off frequency of that filter is f_c. Assume that we have $f_0 \ll f_c$.

   a) Express $Y(f)$ in terms of $M(f)$.
   b) Express $Z(f)$ in terms of $M(f)$ for $\phi = 0$.
   c) Express $Z(f)$ in terms of $M(f)$ for $\phi = \pi/2$. You may want to use that $\sin(x + \pi/2) = \cos(x)$.
   d) Interpret the results!

(5 p)

Equation Service

Below, E denotes *average* signal energy.

Snell's law:	$n_1 \sin\theta_1 = n_2 \sin\theta_2$
Convolution:	$(a * b)(t) \triangleq \int\limits_{-\infty}^{\infty} a(\tau)b(t-\tau)d\tau$
Fourier transform:	$\mathcal{F}\left\{x(t)\right\} \triangleq \int\limits_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$
Inverse Fourier transform:	$\mathcal{F}^{-1}\left\{X(f)\right\} \triangleq \int\limits_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$
Parseval's relation:	$\int\limits_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \int\limits_{-\infty}^{\infty} \lvert X(f)\rvert^2 df$
Derivatives:	$\mathcal{F}\left\{\frac{d^n}{dt^n}x(t)\right\} = (j2\pi f)^n X(f)$
Stationary cosine:	$\mathcal{F}\left\{\cos\left(2\pi f_{\mathrm{c}}t\right)\right\} = \frac{1}{2}\left(\delta(f+f_{\mathrm{c}}) + \delta(f-f_{\mathrm{c}})\right)$
Stationary sine:	$\mathcal{F}\left\{\sin\left(2\pi f_{\mathrm{c}}t\right)\right\} = \frac{j}{2}\left(\delta(f+f_{\mathrm{c}}) - \delta(f-f_{\mathrm{c}})\right)$
Time-discrete Fourier transform:	$\mathcal{F}\left\{x[n]\right\} \triangleq \sum\limits_{n=-\infty}^{\infty} x[n]e^{-j2\pi\theta n}$
Time-discrete Inverse Fourier transform:	$\mathcal{F}^{-1}\left\{X[\theta]\right\} \triangleq \int\limits_{-1/2}^{1/2} X[\theta]e^{j2\pi\theta n}d\theta$
Poisson's summation formula:	$\tilde{X}[\theta] = \frac{1}{T}\sum\limits_{k=-\infty}^{\infty} X\left(\frac{\theta-k}{T}\right)$
Pulse Amplitude Modulation:	$Y(f) = P(f)\tilde{X}[fT]$
OOK & BFSK:	$P_{\mathrm{e}} = Q\left(\sqrt{\frac{E}{N_0}}\right)$
BPSK:	$P_{\mathrm{e}} = Q\left(\sqrt{\frac{2E}{N_0}}\right)$
4-ASK:	$P_{\mathrm{e}} = \frac{3}{2}Q\left(\sqrt{\frac{2E}{5N_0}}\right)$
M-PSK ($M > 2$):	$P_{\mathrm{e}} \approx 2Q\left(\sqrt{\frac{2E}{N_0}}\sin\frac{\pi}{M}\right)$
16-QAM:	$P_{\mathrm{e}} \approx 3Q\left(\sqrt{\frac{E}{5N_0}}\right)$
M-FSK:	$P_{\mathrm{e}} \approx (M-1)Q\left(\sqrt{\frac{E}{N_0}}\right)$
Std integral:	$\int \frac{1}{1+x^2}dx = \arctan(x) + C$
Krafts inequality:	$\sum\limits_i 2^{-l_i} \leq 1$

The Q-function, table of $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ for $0.00 \le x \le 5.99$.

x	0	1	2	3	4	5	6	7	8	9	exp
0.0	5.0000	4.9601	4.9202	4.8803	4.8405	4.8006	4.7608	4.7210	4.6812	4.6414	
0.1	4.6017	4.5620	4.5224	4.4828	4.4433	4.4038	4.3644	4.3251	4.2858	4.2465	
0.2	4.2074	4.1683	4.1294	4.0905	4.0517	4.0129	3.9743	3.9358	3.8974	3.8591	
0.3	3.8209	3.7828	3.7448	3.7070	3.6693	3.6317	3.5942	3.5569	3.5197	3.4827	
0.4	3.4458	3.4090	3.3724	3.3360	3.2997	3.2636	3.2276	3.1918	3.1561	3.1207	
0.5	3.0854	3.0503	3.0153	2.9806	2.9460	2.9116	2.8774	2.8434	2.8096	2.7760	−1
0.6	2.7425	2.7093	2.6763	2.6435	2.6109	2.5785	2.5463	2.5143	2.4825	2.4510	
0.7	2.4196	2.3885	2.3576	2.3270	2.2965	2.2663	2.2363	2.2065	2.1770	2.1476	
0.8	2.1186	2.0897	2.0611	2.0327	2.0045	1.9766	1.9489	1.9215	1.8943	1.8673	
0.9	1.8406	1.8141	1.7879	1.7619	1.7361	1.7106	1.6853	1.6602	1.6354	1.6109	
1.0	1.5866	1.5625	1.5386	1.5151	1.4917	1.4686	1.4457	1.4231	1.4007	1.3786	
1.1	1.3567	1.3350	1.3136	1.2924	1.2714	1.2507	1.2302	1.2100	1.1900	1.1702	
1.2	1.1507	1.1314	1.1123	1.0935	1.0749	1.0565	1.0383	1.0204	1.0027	9.8525	
1.3	9.6800	9.5098	9.3418	9.1759	9.0123	8.8508	8.6915	8.5343	8.3793	8.2264	
1.4	8.0757	7.9270	7.7804	7.6359	7.4934	7.3529	7.2145	7.0781	6.9437	6.8112	
1.5	6.6807	6.5522	6.4255	6.3008	6.1780	6.0571	5.9380	5.8208	5.7053	5.5917	
1.6	5.4799	5.3699	5.2616	5.1551	5.0503	4.9471	4.8457	4.7460	4.6479	4.5514	
1.7	4.4565	4.3633	4.2716	4.1815	4.0930	4.0059	3.9204	3.8364	3.7538	3.6727	−2
1.8	3.5930	3.5148	3.4380	3.3625	3.2884	3.2157	3.1443	3.0742	3.0054	2.9379	
1.9	2.8717	2.8067	2.7429	2.6803	2.6190	2.5588	2.4998	2.4419	2.3852	2.3295	
2.0	2.2750	2.2216	2.1692	2.1178	2.0675	2.0182	1.9699	1.9226	1.8763	1.8309	
2.1	1.7864	1.7429	1.7003	1.6586	1.6177	1.5778	1.5386	1.5003	1.4629	1.4262	
2.2	1.3903	1.3553	1.3209	1.2874	1.2545	1.2224	1.1911	1.1604	1.1304	1.1011	
2.3	1.0724	1.0444	1.0170	9.9031	9.6419	9.3867	9.1375	8.8940	8.6563	8.4242	
2.4	8.1975	7.9763	7.7603	7.5494	7.3436	7.1428	6.9469	6.7557	6.5691	6.3872	
2.5	6.2097	6.0366	5.8677	5.7031	5.5426	5.3861	5.2336	5.0849	4.9400	4.7988	
2.6	4.6612	4.5271	4.3965	4.2692	4.1453	4.0246	3.9070	3.7926	3.6811	3.5726	−3
2.7	3.4670	3.3642	3.2641	3.1667	3.0720	2.9798	2.8901	2.8028	2.7179	2.6354	
2.8	2.5551	2.4771	2.4012	2.3274	2.2557	2.1860	2.1182	2.0524	1.9884	1.9262	
2.9	1.8658	1.8071	1.7502	1.6948	1.6411	1.5889	1.5382	1.4890	1.4412	1.3949	
3.0	1.3499	1.3062	1.2639	1.2228	1.1829	1.1442	1.1067	1.0703	1.0350	1.0008	
3.1	9.6760	9.3544	9.0426	8.7403	8.4474	8.1635	7.8885	7.6219	7.3638	7.1136	
3.2	6.8714	6.6367	6.4095	6.1895	5.9765	5.7703	5.5706	5.3774	5.1904	5.0094	
3.3	4.8342	4.6648	4.5009	4.3423	4.1889	4.0406	3.8971	3.7584	3.6243	3.4946	−4
3.4	3.3693	3.2481	3.1311	3.0179	2.9086	2.8029	2.7009	2.6023	2.5071	2.4151	
3.5	2.3263	2.2405	2.1577	2.0778	2.0006	1.9262	1.8543	1.7849	1.7180	1.6534	
3.6	1.5911	1.5310	1.4730	1.4171	1.3632	1.3112	1.2611	1.2128	1.1662	1.1213	
3.7	1.0780	1.0363	9.9611	9.5740	9.2010	8.8417	8.4957	8.1624	7.8414	7.5324	
3.8	7.2348	6.9483	6.6726	6.4072	6.1517	5.9059	5.6694	5.4418	5.2228	5.0122	
3.9	4.8096	4.6148	4.4274	4.2473	4.0741	3.9076	3.7475	3.5936	3.4458	3.3037	−5
4.0	3.1671	3.0359	2.9099	2.7888	2.6726	2.5609	2.4536	2.3507	2.2518	2.1569	
4.1	2.0658	1.9783	1.8944	1.8138	1.7365	1.6624	1.5912	1.5230	1.4575	1.3948	
4.2	1.3346	1.2769	1.2215	1.1685	1.1176	1.0689	1.0221	9.7736	9.3447	8.9337	
4.3	8.5399	8.1627	7.8015	7.4555	7.1241	6.8069	6.5031	6.2123	5.9340	5.6675	
4.4	5.4125	5.1685	4.9350	4.7117	4.4979	4.2935	4.0980	3.9110	3.7322	3.5612	−6
4.5	3.3977	3.2414	3.0920	2.9492	2.8127	2.6823	2.5577	2.4386	2.3249	2.2162	
4.6	2.1125	2.0133	1.9187	1.8283	1.7420	1.6597	1.5810	1.5060	1.4344	1.3660	
4.7	1.3008	1.2386	1.1792	1.1226	1.0686	1.0171	9.6796	9.2113	8.7648	8.3391	
4.8	7.9333	7.5465	7.1779	6.8267	6.4920	6.1731	5.8693	5.5799	5.3043	5.0418	
4.9	4.7918	4.5538	4.3272	4.1115	3.9061	3.7107	3.5247	3.3476	3.1792	3.0190	−7
5.0	2.8665	2.7215	2.5836	2.4524	2.3277	2.2091	2.0963	1.9891	1.8872	1.7903	
5.1	1.6983	1.6108	1.5277	1.4487	1.3737	1.3024	1.2347	1.1705	1.1094	1.0515	
5.2	9.9644	9.4420	8.9462	8.4755	8.0288	7.6050	7.2028	6.8212	6.4592	6.1158	
5.3	5.7901	5.4813	5.1884	4.9106	4.6473	4.3977	4.1611	3.9368	3.7243	3.5229	−8
5.4	3.3320	3.1512	2.9800	2.8177	2.6640	2.5185	2.3807	2.2502	2.1266	2.0097	
5.5	1.8990	1.7942	1.6950	1.6012	1.5124	1.4283	1.3489	1.2737	1.2026	1.1353	
5.6	1.0718	1.0116	9.5479	9.0105	8.5025	8.0224	7.5686	7.1399	6.7347	6.3520	
5.7	5.9904	5.6488	5.3262	5.0215	4.7338	4.4622	4.2057	3.9636	3.7350	3.5193	−9
5.8	3.3157	3.1236	2.9424	2.7714	2.6100	2.4579	2.3143	2.1790	2.0513	1.9310	
5.9	1.8175	1.7105	1.6097	1.5147	1.4251	1.3407	1.2612	1.1863	1.1157	1.0492	

For $x > 0$, we have $(1 - x^{-2}) \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt < Q(x) < \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$. For large x we have $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$.