

Exam in TSKS01 Digital Communication

Exam code: TEN1

Date: 2020-08-22 **Time:** 8:00–12:00

Place: Distance exam with special instructions:
Submit your solution as a single PDF document using the submission system in Lisam. The submission is open until 12:30, so you have 30 min to create a PDF. (If the submission system doesn't work, please send the exam solution to the teacher by email before 12:30.)
Write your name and personal number at every page and number the pages. Verify that your PDF is complete since no solutions are accepted after the deadline.

Teacher: Emil Björnson, emil.bjornson@liu.se

Visiting exam: The examiner is available to answer questions on 013 - 28 67 32.

Allowed aids: Due to the distance mode, all aids are allowed but all forms of cooperation with other people during the exam hours is strictly forbidden. We recommend the following aids: Pocket calculator.
Olofsson, Björnson: Introduction to Digital Communication.
Olofsson: Tables and Formulas for Signal Theory.

Number of tasks: 7

Solutions: Will be published after the exam on the course web page.

Result: You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.

Exam return: The corrected exams will be sent back to the students in Lisam.

Important: **Solutions and answers must be given in English.**

Grading: This exam consists of three parts: an introductory task, a question part, and a problem-solving part. The introductory task consists of two rather simple subtasks that test the ability to perform standard calculations. Each task in the question part and the problem-solving part can give the number of points indicated in the margin. The question part can give you at most 10 points and the problem-solving part can give you at most 20 points. For passing the exam, you need

- at least one of the two subtasks of the introductory task solved correctly,
- at least 3 points from the question part,
- at least 6 points from the problem-solving part,
- and totally at least 14 points.

Grade limits:

- Grade three (ECTS C): 14 points,
- Grade four (ECTS B): 19 points,
- Grade five (ECTS A): 24 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

Introductory task

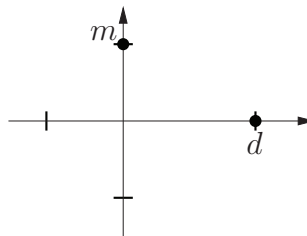
1 This task has to be solved correctly as partial fulfillment for passing the exam.

a. A binary linear block code has the following generator matrix:

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Determine all the codewords.

b. A binary modulation scheme uses the following two signal points.



Let m be the month that you were born (e.g., August means $m = 8$) and let d be the day of the month that you were born (e.g., $d = 18$). The numbers must coincide with those in your personal number.

Determine the error probability if we communicate over an AWGN channel where the noise has power spectral density $N_0/2 = 3$. The receiver uses an ML detector. Use the table on the last page of the exam when computing the final answer.

Question part

- 2 Answer the following questions with 1-5 sentences each. You must write the sentences on your own to show that you can explain it properly. Do not copy any statements from the textbook or other sources. (5p)
- a. According to the course book, the symbol error probability with BPSK is $P_e = Q(\sqrt{2E_{\text{avg}}/N_0})$, while the symbol error probability with M-PSK is $P_e \approx 2Q(\sqrt{2E_{\text{avg}}/N_0} \sin(\pi/M))$. Why is there an equality in the first case and an approximation sign in the second case?
 - b. Give an example of a situation in digital communications when the Gram-Schmidt orthogonalization procedure can be useful.
 - c. In Figure 9.3, we can see that the 16-QAM curve with Hamming coding is mainly to the left of the uncoded 16-QAM curve, but the two curves switch order at around 20 dB. Why does this happen?
 - d. What is the difference between hard and soft decoding of a linear block code?
 - e. Decision rule 5.5 (on page 86) says that the ML estimate is the constellation point that is closest to the received signal. Why is this ML decision rule not affected by the noise variance?

3 Are the following claims true or false? You do not need to explain your answers. (5p)

- a. The nearest neighbor approximation will always give a smaller value than the true error probability.
- b. There exist linear block codes of length n that have a minimum distance d that is equal to the length: $d = n$.
- c. Decision rule 8.4 (on page 165) is based on the assumption $p < 0.5$. If we instead consider $p > 0.5$, then the decision rule becomes

$$\hat{c} = \operatorname{argmax}_{\bar{c} \in \mathcal{C}} d_{\text{H}}(\bar{x}, \bar{c}).$$

- d. The matched filter receiver is preferred over the correlation receiver since it gives a smaller symbol error probability.
- e. The reason that the symbol error probabilities contain the Q -function is that we have Gaussian noise. If we would have had another noise distribution, other functions would appear.

For each of the claims above, a correct answer gives you +1 point, while an incorrect answer gives you -1 point. No answer give you 0 points for that claim, so a good strategy is to only give an answer if you are sure that it is correct. You cannot get less than 0 points totally from this task.

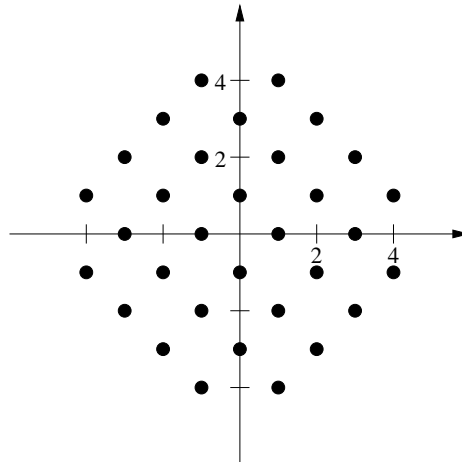
Problem-solving part

- 4 A binary linear block code has the following generator matrix: (5 p)

$$G = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

- a. Determine the length, dimension, and size of the code. (1p)
 - b. Determine an alternative generator matrix that is on systematic form. You need to explain step-by-step how you obtain it. (2p)
 - c. Prove whether or not this code satisfies the Singleton bound with equality. (2p)
- 5 We want to send the message $m(x) = x^5 + x^3 + x$ using a CRC code defined by the CRC polynomial $p(x) = x^4 + x + 1$. (5p)
- a. Which codeword $c(x) = x^{n-k}m(x) + r(x)$ corresponds to $m(x)$? Include a computation of $r(x)$ based polynomial division. (2p)
 - b. Draw a linear feedback shift register for CRC code generation. (1p)
 - c. Show step-by-step how the remainder $r(x)$ can be obtained by using the linear feedback shift register from Part **b**. (2p)

- 6 Consider the following constellation diagram, where all the points are equally probable: (5 p)

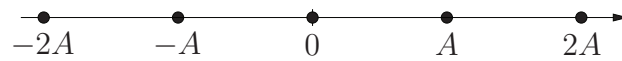


The transmission is done over an AWGN channel with power spectral density $N_0/2$.

- a. Give an expression of the symbol error probability based on the nearest neighbor approximation.
- b. Compute the average energy, E_{avg} , of the constellation.
(Hint: The points are at five different distances from the origin.)
- c. For which value of the signal-to-noise ratio E_{avg}/N_0 will the symbol error probability be 10^{-2} ?

7 Consider the following signal space diagram:

(5 p)



The transmission is done over an AWGN channel with power spectral density $N_0/2$.

- a. Use the nearest neighbor approximation to compute an expression for the symbol error probability in terms of A and N_0 . (1p)
- b. Use the union bound to compute an expression for the symbol error probability in terms of A and N_0 . (2p)
- c. Compute the *exact* symbol error probability. Express the result in terms of the average energy per symbol and the maximum energy per symbol. (2p)

The Q -function, table of $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ for $0.00 \leq x \leq 5.99$.

x	0	1	2	3	4	5	6	7	8	9	exp
0.0	5.0000	4.9601	4.9202	4.8803	4.8405	4.8006	4.7608	4.7210	4.6812	4.6414	-1
0.1	4.6017	4.5620	4.5224	4.4828	4.4433	4.4038	4.3644	4.3251	4.2858	4.2465	
0.2	4.2074	4.1683	4.1294	4.0905	4.0517	4.0129	3.9743	3.9358	3.8974	3.8591	
0.3	3.8209	3.7828	3.7448	3.7070	3.6693	3.6317	3.5942	3.5569	3.5197	3.4827	
0.4	3.4458	3.4090	3.3724	3.3360	3.2997	3.2636	3.2276	3.1918	3.1561	3.1207	
0.5	3.0854	3.0503	3.0153	2.9806	2.9460	2.9116	2.8774	2.8434	2.8096	2.7760	
0.6	2.7425	2.7093	2.6763	2.6435	2.6109	2.5785	2.5463	2.5143	2.4825	2.4510	
0.7	2.4196	2.3885	2.3576	2.3270	2.2965	2.2663	2.2363	2.2065	2.1770	2.1476	
0.8	2.1186	2.0897	2.0611	2.0327	2.0045	1.9766	1.9489	1.9215	1.8943	1.8673	
0.9	1.8406	1.8141	1.7879	1.7619	1.7361	1.7106	1.6853	1.6602	1.6354	1.6109	
1.0	1.5866	1.5625	1.5386	1.5151	1.4917	1.4686	1.4457	1.4231	1.4007	1.3786	
1.1	1.3567	1.3350	1.3136	1.2924	1.2714	1.2507	1.2302	1.2100	1.1900	1.1702	
1.2	1.1507	1.1314	1.1123	1.0935	1.0749	1.0565	1.0383	1.0204	1.0027	9.8525	
1.3	9.6800	9.5098	9.3418	9.1759	9.0123	8.8508	8.6915	8.5343	8.3793	8.2264	
1.4	8.0757	7.9270	7.7804	7.6359	7.4934	7.3529	7.2145	7.0781	6.9437	6.8112	
1.5	6.6807	6.5522	6.4255	6.3008	6.1780	6.0571	5.9380	5.8208	5.7053	5.5917	
1.6	5.4799	5.3699	5.2616	5.1551	5.0503	4.9471	4.8457	4.7460	4.6479	4.5514	
1.7	4.4565	4.3633	4.2716	4.1815	4.0930	4.0059	3.9204	3.8364	3.7538	3.6727	
1.8	3.5930	3.5148	3.4380	3.3625	3.2884	3.2157	3.1443	3.0742	3.0054	2.9379	
1.9	2.8717	2.8067	2.7429	2.6803	2.6190	2.5588	2.4998	2.4419	2.3852	2.3295	
2.0	2.2750	2.2216	2.1692	2.1178	2.0675	2.0182	1.9699	1.9226	1.8763	1.8309	
2.1	1.7864	1.7429	1.7003	1.6586	1.6177	1.5778	1.5386	1.5003	1.4629	1.4262	
2.2	1.3903	1.3553	1.3209	1.2874	1.2545	1.2224	1.1911	1.1604	1.1304	1.1011	
2.3	1.0724	1.0444	1.0170	9.9031	9.6419	9.3867	9.1375	8.8940	8.6563	8.4242	
2.4	8.1975	7.9763	7.7603	7.5494	7.3436	7.1428	6.9469	6.7557	6.5691	6.3872	
2.5	6.2097	6.0366	5.8677	5.7031	5.5426	5.3861	5.2336	5.0849	4.9400	4.7988	
2.6	4.6612	4.5271	4.3965	4.2692	4.1453	4.0246	3.9070	3.7926	3.6811	3.5726	
2.7	3.4670	3.3642	3.2641	3.1667	3.0720	2.9798	2.8901	2.8028	2.7179	2.6354	
2.8	2.5551	2.4771	2.4012	2.3274	2.2557	2.1860	2.1182	2.0524	1.9884	1.9262	
2.9	1.8658	1.8071	1.7502	1.6948	1.6411	1.5889	1.5382	1.4890	1.4412	1.3949	
3.0	1.3499	1.3062	1.2639	1.2228	1.1829	1.1442	1.1067	1.0703	1.0350	1.0008	
3.1	9.6760	9.3544	9.0426	8.7403	8.4474	8.1635	7.8885	7.6219	7.3638	7.1136	
3.2	6.8714	6.6367	6.4095	6.1895	5.9765	5.7703	5.5706	5.3774	5.1904	5.0094	
3.3	4.8342	4.6648	4.5009	4.3423	4.1889	4.0406	3.8971	3.7584	3.6243	3.4946	
3.4	3.3693	3.2481	3.1311	3.0179	2.9086	2.8029	2.7009	2.6023	2.5071	2.4151	
3.5	2.3263	2.2405	2.1577	2.0778	2.0006	1.9262	1.8543	1.7849	1.7180	1.6534	
3.6	1.5911	1.5310	1.4730	1.4171	1.3632	1.3112	1.2611	1.2128	1.1662	1.1213	
3.7	1.0780	1.0363	9.9611	9.5740	9.2010	8.8417	8.4957	8.1624	7.8414	7.5324	
3.8	7.2348	6.9483	6.6726	6.4072	6.1517	5.9059	5.6694	5.4418	5.2228	5.0122	
3.9	4.8096	4.6148	4.4274	4.2473	4.0741	3.9076	3.7475	3.5936	3.4458	3.3037	
4.0	3.1671	3.0359	2.9099	2.7888	2.6726	2.5609	2.4536	2.3507	2.2518	2.1569	
4.1	2.0658	1.9783	1.8944	1.8138	1.7365	1.6624	1.5912	1.5230	1.4575	1.3948	
4.2	1.3346	1.2769	1.2215	1.1685	1.1176	1.0689	1.0221	9.7736	9.3447	8.9337	
4.3	8.5399	8.1627	7.8015	7.4555	7.1241	6.8069	6.5031	6.2123	5.9340	5.6675	
4.4	5.4125	5.1685	4.9350	4.7117	4.4979	4.2935	4.0980	3.9110	3.7322	3.5612	
4.5	3.3977	3.2414	3.0920	2.9492	2.8127	2.6823	2.5577	2.4386	2.3249	2.2162	
4.6	2.1125	2.0133	1.9187	1.8283	1.7420	1.6597	1.5810	1.5060	1.4344	1.3660	
4.7	1.3008	1.2386	1.1792	1.1226	1.0686	1.0171	9.6796	9.2113	8.7648	8.3391	
4.8	7.9333	7.5465	7.1779	6.8267	6.4920	6.1731	5.8693	5.5799	5.3043	5.0418	
4.9	4.7918	4.5538	4.3272	4.1115	3.9061	3.7107	3.5247	3.3476	3.1792	3.0190	
5.0	2.8665	2.7215	2.5836	2.4524	2.3277	2.2091	2.0963	1.9891	1.8872	1.7903	
5.1	1.6983	1.6108	1.5277	1.4487	1.3737	1.3024	1.2347	1.1705	1.1094	1.0515	
5.2	9.9644	9.4420	8.9462	8.4755	8.0288	7.6050	7.2028	6.8212	6.4592	6.1158	
5.3	5.7901	5.4813	5.1884	4.9106	4.6473	4.3977	4.1611	3.9368	3.7243	3.5229	
5.4	3.3320	3.1512	2.9800	2.8177	2.6640	2.5185	2.3807	2.2502	2.1266	2.0097	
5.5	1.8990	1.7942	1.6950	1.6012	1.5124	1.4283	1.3489	1.2737	1.2026	1.1353	
5.6	1.0718	1.0116	9.5479	9.0105	8.5025	8.0224	7.5686	7.1399	6.7347	6.3520	
5.7	5.9904	5.6488	5.3262	5.0215	4.7338	4.4622	4.2057	3.9636	3.7350	3.5193	
5.8	3.3157	3.1236	2.9424	2.7714	2.6100	2.4579	2.3143	2.1790	2.0513	1.9310	
5.9	1.8175	1.7105	1.6097	1.5147	1.4251	1.3407	1.2612	1.1863	1.1157	1.0492	

For $x > 0$, we have $(1 - x^{-2}) \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt < Q(x) < \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$. For large x we have $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$.