TSKS01 Digital Communication

Solutions for the exam 2019-08-24

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Introductory task

As partial fulfillment to pass the exam, you have to solve at least one of these two subtasks correctly.

1

a. The distance between the two signal points is $d = \sqrt{3^2 + 4^2} = 5$ and then the error probability is

$$P_\mathrm{e} = Q\left(\frac{d}{\sqrt{2N_0}}\right) = Q\left(\frac{5}{\sqrt{2}}\right) \approx Q(3.54) \approx 2.0006 \cdot 10^{-4}.$$

b. The codewords are generated by multiplying G with all length-two binary vectors. This generates the codewords 0 0 0 0 0, 1 0 1 1 0, 0 1 0 1 1, and 1 1 1 0 1.

Question part

The questions in this part can give you at most 5 points each. You need at least 3 points from this part of the exam to pass.

2

The answer should give a reasonable description of syndrome decoding Here are some main points that give one point each:

- Dimension of parity check matrix and received signal.
- Explanation of what happens when a received signal with error is multiplied with the parity-check matrix.
- Explanation of how to identify the error from the syndrome.
- Motivation of why syndrome decoding leads to an ML detector
- An example of syndrome decoding with numbers in the matrices/vectors.

3

a. **True**, a symbol error normally leads to some correct and incorrect bits. In binary modulations the symbol and bit error probabilities are equal.

b. **True**, the linearity implies that the message with only zeros is always mapped to a codeword with only zeros.

c. **False**, it is always smaller or equal to one.

d. **True**, the eye pattern is determined by the basis function. Some basis functions are more affected by sampling instance mismatches than others.

e. **False**, it is the phase that carries the information in PSK, while all signal points have the same amplitude.

Problem-solving part

The problems in this part can give you at most 5 points each. You need at least 6 points from this part of the exam to pass.

4

Let P_{e} denote the symbol-error probability and let P_{b} denote the bit-error probability.

a. In 16-PSK, we can use a Gray code, so that errors to nearest neighbours always result in one bit-error among the 4 bits. Thus, a nearest-neighbour approximation of the relation between P_{b} and P_{e} is

$$P_{\mathrm{b}} \approx P_{\mathrm{e}}/4.$$

b. In 16-FSK, all signals are on the same distance from all other signals. Thus, the allocation of the bit patterns on the signals is irrelevant. There is no use for a Gray code here. Given that we do have an error, any of the 15 error patterns

$$\begin{array}{c} 0001 \\ 0010 \\ 0011 \\ \vdots \\ 1101 \\ 1110 \\ 1111 \end{array}$$

are equally probable, where 1 should be interpreted as an erroneous bit. Each column in that table contains 8 ones. That means that in 8 out of 15 cases, we get an error in any given bit. The bit error probability is therefore given by

$$P_{\mathrm{b}} = \frac{8}{15} P_{\mathrm{e}}.$$

Notice that this is not an approximation.

Answer: a. $P_{\mathrm{b}} \approx P_{\mathrm{e}}/4$, **b.** $P_{\mathrm{b}} = \frac{8}{15} P_{\mathrm{e}}$.

5

a. The feedback shift register for this CRC code is given in Figure 8.8.

b. The first step in the encoding is to compute $x^{n-k}m(x)$ which in this case is $x^4(x^5 + x^2 + x) = x^9 + x^6 + x^5$ since $n - k = 4$. Next, we should divide $x^{n-k}m(x)$ by the CRC polynomial, which leads to the remainder $r(x) = x^2$. Finally, the codeword is formed as

$$c(x) = x^{n-k}m(x) + r(x) = x^9 + x^6 + x^5 + x^2.$$

One can also use the feedback shift register from Part a to compute that the remainder is x^2.

Answer: a. See Figure 8.8 in the course book.
b. The codeword $c(x) = x^9 + x^6 + x^5 + x^2$.

6

Each codeword in a Hamming-$[7, 4]$ code has length 7. We increase that by one when we append one bit to every codeword. Thus, we have the length $n = 8$. We do not change the size of the code, since there is one codeword in the new code for each codeword in the Hamming code. So, we have the same dimension as for the original code, $k = 4$. The size is the number of codewords, $M = 2^k = 16$. For the minimum distance, first we observe that the appended bit is given by a linear mapping from the original codeword. Thus, the resulting code is linear since the original code is linear. So, we can use the smallest non-zero weight of the code as the minimum distance. The new bit is 0 if the original codeword has even weight and it is otherwise 1. Thus, if the original codeword has even weight, then the resulting codeword has the same weight. But, if the original codeword has odd weight, then the weight is increased by one. The minimum distance of the Hamming code is 3, so the smallest weight among codewords is 3 in the Hamming code. Those codewords will be appended by 1, so that the corresponding codewords in the resulting code will have weight 4. Consequently, the minimum distance of the resulting code is $d = 4$.

Answer: $n = 8$, $k = 4$, $M = 16$, $d = 4$.

7

a. The packet error probability is $1 - \left(1 - Q\left(\sqrt{\frac{2E_\mathrm{b}}{N_0}}\right)\right)^{11}$. This equals 0.05 if $Q\left(\sqrt{\frac{2E_\mathrm{b}}{N_0}}\right) \approx 4.65 \cdot 10^{-3}$, leading to $E_\mathrm{b}/N_0 \approx 3.38$. Hence, we need $E_\mathrm{b}/N_0 \geq 3.38$.

b. The minimum distance is 3, so the code can correct 1 error.

c. We can apply (9.1) for $d = 3$ and $L_p = 15$ to get

$$1 - (1 - p)^{15} - 15p\,(1 - p)^{14}$$

where $p = Q\left(\sqrt{\frac{2E}{N_0}}\right)$.

d. We need to set $E = E_\mathrm{b}\frac{11}{15}$ to get the correct energy per bit. By evaluating the expression from Part c, we get 0.016 as the packet error probability.

Answer: See above.