Exam in TSKS01 Digital Communication

Exam code:	TEN1
Date:	2019-04-23 **Time:** 14:00–18:00
Place:	TER3
Teacher:	Emil Björnson, tel: 013 - 28 67 32
Visiting exam:	Around 15:00 and 16:00
Administrator:	Carina Lindström, 013 - 28 44 23, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Pocket calculator with empty memory. Olofsson: Tables and Formulas for Signal Theory.
Number of tasks:	7
Solutions:	Will be published within one week after the exam at http://www.commsys.isy.liu.se/TSKS01
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2019-05-07, 12:45–13:00, Emil Björnson's office, Building B, top floor, corridor A between entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: **Solutions and answers must be given in English.**

Grading: This exam consists of three parts: an introductory task, a question part, and a problem-solving part. The introductory task consists of two rather simple subtasks that test the ability to perform standard calculations. Each task in the question part and the problem-solving part can give the number of points indicated in the margin. The question part can give you at most 10 points and the problem-solving part can give you at most 20 points. For passing the exam, you need

- at least one of the two subtasks of the introductory task solved correctly,
- at least 3 points from the question part,
- at least 6 points from the problem-solving part,
- and totally at least 14 points.

Grade limits:

- Grade three (ECTS C): 14 points,
- Grade four (ECTS B): 19 points,
- Grade five (ECTS A): 24 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

Introductory task

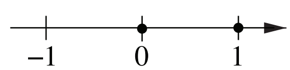
1 You need to solve at least one of these subtasks correctly as partial fulfillment for passing the exam.

a. A linear block code has the following generator matrix:

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}.$$

What is the length of this code?

b. Consider the following binary modulation scheme:



Determine the error probability if we communicate over an AWGN channel where the noise has power spectral density $N_0 = 1$. The receiver uses an ML detector.

Question part

2 Explain the concept of link adaptation and when it is useful in digital communication systems. Make sure to describe the throughput formula and what design choices that affect it. Draw an example figure with throughput on the vertical axis and SNR on the horizontal axis. The figure should contain three different combinations of modulation/coding, and you need to explain when each combination is desirable. (5 p)

3 Are the following claims true or false? You do not need to explain your answers. (5p)

a. BFSK and OOK have the same symbol error probability, if the average signal energy and noise variance are the same.

b. Hard decoding of a linear block code gives a lower error probability than soft decoding.

c. The bandwidth of a pulse-amplitude modulated communication signal is proportional to the symbol time T.

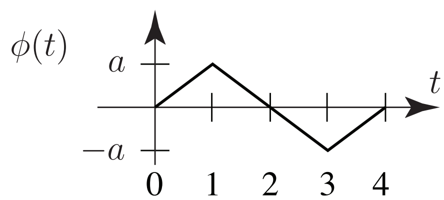
d. The Viterbi algorithm is a sequential implementation of ML detection.

e. Gray coding reduces the symbol error probability.

For each of the claims above, a correct answer gives you $+1$ point, while an incorrect answer gives you -1 point. No answer give you 0 points for that claim, so a good strategy is to only give an answer if you are sure that it is correct. You cannot get less than 0 points totally from this task.

Problem-solving part

4 We communicate over an AWGN channel using BPSK with $s_0(t) = \phi(t)$ and $s_1(t) = -\phi(t)$. The pulse function $\phi(t)$ is defined as: (5 p)



The noise has power spectral density $N_0/2$. Answer the following questions:

a. What is the exact symbol error probability, using ML detection? Write the expression as a function of a and N_0. (2p)

b. We want to achieve an error probability of 10^{-3}. How large must a be to achieve this, assuming $N_0 = 2$? (2p)

c. Suppose the two signals are $s_0(t) = \phi(4t)$ and $s_1(t) = -\phi(4t)$ instead. Derive a new exact symbol error probability expression, using ML detection. Is it smaller, larger, or the same as in **a**? (1p)

5 Hamming codes are linear binary codes with length $n = 2^m - 1$ and dimension $k = 2^m - m - 1$, for some integer $m \geq 2$. The parity-check matrix of a Hamming code contains all non-zero binary vectors of length m. (5 p)

a. Determine a parity check matrix of a Hamming code with $m = 3$.

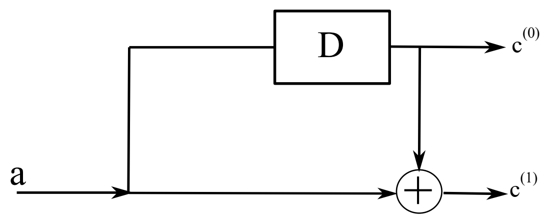
b. Determine a generator matrix of a Hamming code with $m = 3$.

c. Determine the minimum distance of a Hamming code with $m = 3$.

d. Derive an expression for the minimum distance of a Hamming code that holds for any $m \geq 2$.

e. Prove what happens to the rate of the code as $m \to \infty$.

6 Consider the following convolutional encoder: (5 p)

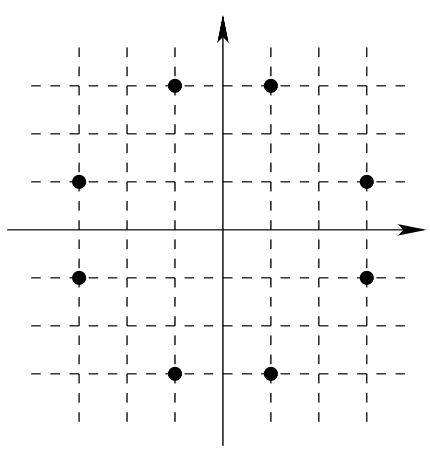


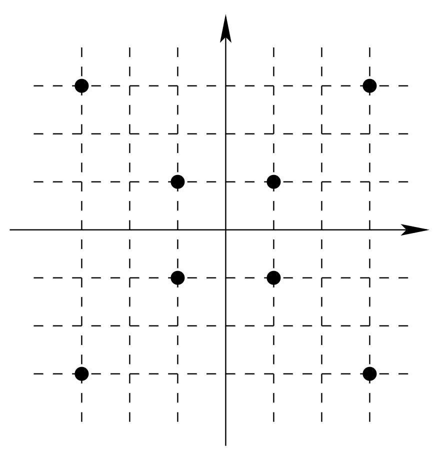
a. Let $g^{(0)}$ and $g^{(1)}$ be the impulse responses for the outputs $c^{(0)}$ and $c^{(0)}$, respectively. Derive the D-transforms of the impulse responses, $G^{(0)}(D)$ and $G^{(1)}(D)$.

b. Draw the state transition diagram of the encoder.

c. Draw a trellis representation for 4 bits and use the Viterbi algorithm to decode the received sequence $r = 11\,11\,11\,11\,11$.
(Hint: The trellis terminates at the zero state.)

7 Consider the following two signal constellations: (5 p)





The signal constellations are assumed to be used for communication over an AWGN channel with high SNR. Which signal constellation is preferred when the

a. average energy E_{avg} is the same for both constellations?

b. maximum energy $E_{\max}$ is the same for both constellations?

The answers must be supported by mathematical expressions.

The Q-function, table of $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ for $0.00 \le x \le 5.99$.

x	0	1	2	3	4	5	6	7	8	9	exp
0.0	5.0000	4.9601	4.9202	4.8803	4.8405	4.8006	4.7608	4.7210	4.6812	4.6414	
0.1	4.6017	4.5620	4.5224	4.4828	4.4433	4.4038	4.3644	4.3251	4.2858	4.2465	
0.2	4.2074	4.1683	4.1294	4.0905	4.0517	4.0129	3.9743	3.9358	3.8974	3.8591	
0.3	3.8209	3.7828	3.7448	3.7070	3.6693	3.6317	3.5942	3.5569	3.5197	3.4827	
0.4	3.4458	3.4090	3.3724	3.3360	3.2997	3.2636	3.2276	3.1918	3.1561	3.1207	
0.5	3.0854	3.0503	3.0153	2.9806	2.9460	2.9116	2.8774	2.8434	2.8096	2.7760	−1
0.6	2.7425	2.7093	2.6763	2.6435	2.6109	2.5785	2.5463	2.5143	2.4825	2.4510	
0.7	2.4196	2.3885	2.3576	2.3270	2.2965	2.2663	2.2363	2.2065	2.1770	2.1476	
0.8	2.1186	2.0897	2.0611	2.0327	2.0045	1.9766	1.9489	1.9215	1.8943	1.8673	
0.9	1.8406	1.8141	1.7879	1.7619	1.7361	1.7106	1.6853	1.6602	1.6354	1.6109	
1.0	1.5866	1.5625	1.5386	1.5151	1.4917	1.4686	1.4457	1.4231	1.4007	1.3786	
1.1	1.3567	1.3350	1.3136	1.2924	1.2714	1.2507	1.2302	1.2100	1.1900	1.1702	
1.2	1.1507	1.1314	1.1123	1.0935	1.0749	1.0565	1.0383	1.0204	1.0027	9.8525	
1.3	9.6800	9.5098	9.3418	9.1759	9.0123	8.8508	8.6915	8.5343	8.3793	8.2264	
1.4	8.0757	7.9270	7.7804	7.6359	7.4934	7.3529	7.2145	7.0781	6.9437	6.8112	
1.5	6.6807	6.5522	6.4255	6.3008	6.1780	6.0571	5.9380	5.8208	5.7053	5.5917	
1.6	5.4799	5.3699	5.2616	5.1551	5.0503	4.9471	4.8457	4.7460	4.6479	4.5514	
1.7	4.4565	4.3633	4.2716	4.1815	4.0930	4.0059	3.9204	3.8364	3.7538	3.6727	−2
1.8	3.5930	3.5148	3.4380	3.3625	3.2884	3.2157	3.1443	3.0742	3.0054	2.9379	
1.9	2.8717	2.8067	2.7429	2.6803	2.6190	2.5588	2.4998	2.4419	2.3852	2.3295	
2.0	2.2750	2.2216	2.1692	2.1178	2.0675	2.0182	1.9699	1.9226	1.8763	1.8309	
2.1	1.7864	1.7429	1.7003	1.6586	1.6177	1.5778	1.5386	1.5003	1.4629	1.4262	
2.2	1.3903	1.3553	1.3209	1.2874	1.2545	1.2224	1.1911	1.1604	1.1304	1.1011	
2.3	1.0724	1.0444	1.0170	9.9031	9.6419	9.3867	9.1375	8.8940	8.6563	8.4242	
2.4	8.1975	7.9763	7.7603	7.5494	7.3436	7.1428	6.9469	6.7557	6.5691	6.3872	
2.5	6.2097	6.0366	5.8677	5.7031	5.5426	5.3861	5.2336	5.0849	4.9400	4.7988	
2.6	4.6612	4.5271	4.3965	4.2692	4.1453	4.0246	3.9070	3.7926	3.6811	3.5726	−3
2.7	3.4670	3.3642	3.2641	3.1667	3.0720	2.9798	2.8901	2.8028	2.7179	2.6354	
2.8	2.5551	2.4771	2.4012	2.3274	2.2557	2.1860	2.1182	2.0524	1.9884	1.9262	
2.9	1.8658	1.8071	1.7502	1.6948	1.6411	1.5889	1.5382	1.4890	1.4412	1.3949	
3.0	1.3499	1.3062	1.2639	1.2228	1.1829	1.1442	1.1067	1.0703	1.0350	1.0008	
3.1	9.6760	9.3544	9.0426	8.7403	8.4474	8.1635	7.8885	7.6219	7.3638	7.1136	
3.2	6.8714	6.6367	6.4095	6.1895	5.9765	5.7703	5.5706	5.3774	5.1904	5.0094	
3.3	4.8342	4.6648	4.5009	4.3423	4.1889	4.0406	3.8971	3.7584	3.6243	3.4946	−4
3.4	3.3693	3.2481	3.1311	3.0179	2.9086	2.8029	2.7009	2.6023	2.5071	2.4151	
3.5	2.3263	2.2405	2.1577	2.0778	2.0006	1.9262	1.8543	1.7849	1.7180	1.6534	
3.6	1.5911	1.5310	1.4730	1.4171	1.3632	1.3112	1.2611	1.2128	1.1662	1.1213	
3.7	1.0780	1.0363	9.9611	9.5740	9.2010	8.8417	8.4957	8.1624	7.8414	7.5324	
3.8	7.2348	6.9483	6.6726	6.4072	6.1517	5.9059	5.6694	5.4418	5.2228	5.0122	
3.9	4.8096	4.6148	4.4274	4.2473	4.0741	3.9076	3.7475	3.5936	3.4458	3.3037	−5
4.0	3.1671	3.0359	2.9099	2.7888	2.6726	2.5609	2.4536	2.3507	2.2518	2.1569	
4.1	2.0658	1.9783	1.8944	1.8138	1.7365	1.6624	1.5912	1.5230	1.4575	1.3948	
4.2	1.3346	1.2769	1.2215	1.1685	1.1176	1.0689	1.0221	9.7736	9.3447	8.9337	
4.3	8.5399	8.1627	7.8015	7.4555	7.1241	6.8069	6.5031	6.2123	5.9340	5.6675	
4.4	5.4125	5.1685	4.9350	4.7117	4.4979	4.2935	4.0980	3.9110	3.7322	3.5612	−6
4.5	3.3977	3.2414	3.0920	2.9492	2.8127	2.6823	2.5577	2.4386	2.3249	2.2162	
4.6	2.1125	2.0133	1.9187	1.8283	1.7420	1.6597	1.5810	1.5060	1.4344	1.3660	
4.7	1.3008	1.2386	1.1792	1.1226	1.0686	1.0171	9.6796	9.2113	8.7648	8.3391	
4.8	7.9333	7.5465	7.1779	6.8267	6.4920	6.1731	5.8693	5.5799	5.3043	5.0418	
4.9	4.7918	4.5538	4.3272	4.1115	3.9061	3.7107	3.5247	3.3476	3.1792	3.0190	−7
5.0	2.8665	2.7215	2.5836	2.4524	2.3277	2.2091	2.0963	1.9891	1.8872	1.7903	
5.1	1.6983	1.6108	1.5277	1.4487	1.3737	1.3024	1.2347	1.1705	1.1094	1.0515	
5.2	9.9644	9.4420	8.9462	8.4755	8.0288	7.6050	7.2028	6.8212	6.4592	6.1158	
5.3	5.7901	5.4813	5.1884	4.9106	4.6473	4.3977	4.1611	3.9368	3.7243	3.5229	−8
5.4	3.3320	3.1512	2.9800	2.8177	2.6640	2.5185	2.3807	2.2502	2.1266	2.0097	
5.5	1.8990	1.7942	1.6950	1.6012	1.5124	1.4283	1.3489	1.2737	1.2026	1.1353	
5.6	1.0718	1.0116	9.5479	9.0105	8.5025	8.0224	7.5686	7.1399	6.7347	6.3520	
5.7	5.9904	5.6488	5.3262	5.0215	4.7338	4.4622	4.2057	3.9636	3.7350	3.5193	−9
5.8	3.3157	3.1236	2.9424	2.7714	2.6100	2.4579	2.3143	2.1790	2.0513	1.9310	
5.9	1.8175	1.7105	1.6097	1.5147	1.4251	1.3407	1.2612	1.1863	1.1157	1.0492	

For $x > 0$, we have $(1 - x^{-2})\frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt < Q(x) < \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$. For large x we have $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$.