LINKÖPINGS UNIVERSITET

Exam in TSKS01 Digital Communication

Exam code:	TEN1
Date:	2017-08-19 **Time:** 08:00–12:00
Place:	KÅRA
Teacher:	Emil Björnson, tel: 013 - 28 67 32
Visiting exam:	Around 09:30
Administrator:	Carina Lindström, 013 - 28 44 23, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Pocket calculator with empty memory. Olofsson: Tables and Formulas for Signal Theory.
Number of tasks:	7
Solutions:	Will be published within five days after the exam at http://www.commsys.isy.liu.se/TSKS01
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2017-09-05, 12:45–13:00, Emil Björnson's office, Building B, top floor, corridor A between entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of three parts: an introductory task, a question part, and a problem part. The introductory task consists of two rather simple subtasks that test the ability to perform standard calculations. Each task in the question part and the problem part can give the number of points indicated in the margin. The question part can give you at most 10 points and the problem part can give you at most 20 points. For passing the exam, you need

- at least one of the two subtasks of the introductory task solved correctly,
- at least 3 points from the question part,
- at least 6 points from the problem part,
- and totally at least 14 points.

Grade limits:

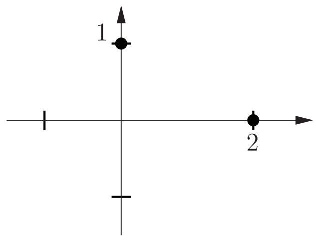
- Grade three (ECTS C): 14 points,
- Grade four (ECTS B): 19 points,
- Grade five (ECTS A): 24 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

Introductory task

1 This task has to be solved correctly as partial fulfillment for passing the exam.

a. A binary modulation scheme uses the following two signal points:



Determine the error probability if we communicate over an AWGN channel where the noise has power spectral density $N_0/2 = 0.5$. The receiver uses an ML detector.

b. Consider a linear code for error control with generator matrix

$$G = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

Compute all the codewords.

Question part

2 A Hamming code is characterized by an integer $m \geq 2$. The parity-check matrix of a Hamming code contains all non-zero binary vectors of length m. (5 p)

Explain as many properties of Hamming codes as you can. For example, what is the dimension and length of such a code, for a given m? What is the minimum distance? Is there any known bound on the error control performance that is satisfied by Hamming codes?

You should also exemplify the generator matrix and parity-check matrix for a Hamming code with $m = 3$.

3 Are the following claims true or false? You do not need to explain your answers. (5 p)

- **a**. The symbol time is proportional to the bandwidth (i.e., larger bandwidth means larger symbol time.)
- **b**. The union bound gives an upper bound on the symbol error probability.
- **c**. CRC codes are primarily used for error detection.
- **d**. The amplitude of the signal carries the information in PSK.
- **e**. Inter-symbol interference appears when the Nyquist condition is satisfied.

For each of the claims above, a correct answer gives you +1 point, while an incorrect answer gives you −1 point. No answer give you 0 points for that claim, so a good strategy is to only give an answer if you are sure that it is correct. You cannot get less than 0 points totally from this task.

Problem part

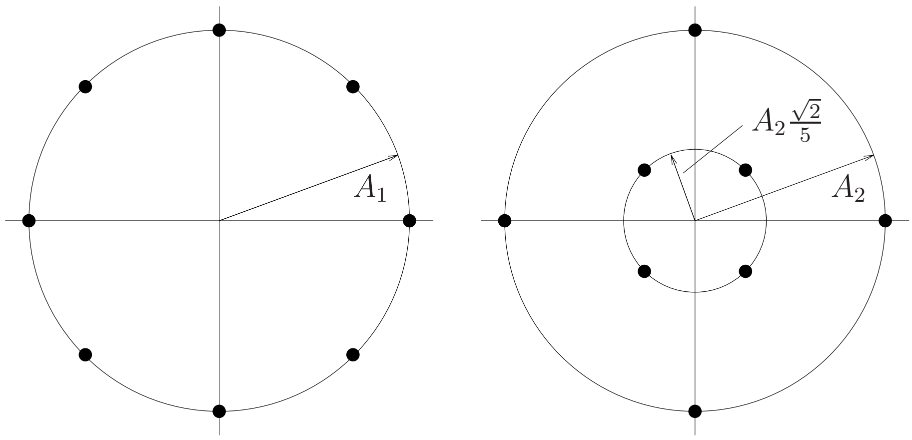
4 A *repetition code* of length n is a code consisting of two codewords only: $c^{(0)} = (0, 0, \ldots, 0)$ and $c^{(1)} = (1, 1, \ldots, 1)$. Suppose n is odd. (5 p)

- **a**. How many errors can be corrected? (1p)
- **b**. Give a parity check matrix H for this code. (2p)
- **c**. Show that the code is *perfect*. (2p)

5 We want to send the message $m(x) = 0 \cdot x^5 + 1 \cdot x^4 + 0 \cdot x^3 + 1 \cdot x^2 + 0 \cdot x + 1 \cdot 1$ using a CRC code defined by the CRC polynomial $p(x) = x^4 + x^3 + x + 1$. (5 p)

- **a**. What are n and k? (2p)
- **b**. Which codeword $c(x)$ corresponds to $m(x)$? (3p)

6 Consider the following two signal space diagrams: (3 p)



a. Determine the symbol error probabilities for these two schemes. It is okay to use an approximation or bound, if you motivate your choice. (3p)

b. Determine the relationship between A_1 and A_2 such that the error probabilities from **a**. are the same. (1p)

c. If A_1 and A_2 are selected to make the average energy equal, which scheme gives the lowest error probability? (1p)

7 The so-called $(\bar{u}, \bar{u}+\bar{v})$ construction is a popular method to construct error control codes from smaller codes. A code $\mathcal{C}$ with parameters $[2n, k_1+k_2, d]$ is constructed based on two linear codes: $\mathcal{C}_1$ with parameters $[n, k_1, d_1]$ and $\mathcal{C}_2$ with parameters $[n, k_2, d_2]$. (5p)

A codeword in $\mathcal{C}$ is created from the two codes by selecting a codeword $\bar{u} \in \mathcal{C}_1$ based on k_1 information bits and selecting a codeword $\bar{v} \in \mathcal{C}_2$ based on k_2 other information bits. The codeword then consists of the n bits in $\bar{u}$ followed by the n bits in $\bar{u}+\bar{v}$, i.e., totally $2n$ bits.

Let $\mathcal{C}_1$ be a simple parity check code with parameters $[4, 3, 2]$ and let $\mathcal{C}_2$ be a repetition code with parameters $[4, 1, 4]$.

a. Determine a generator matrix of the resulting code. (2p)

b. Determine the minimum distance of the code. (3p)

The Q-function, table of $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ for $0.00 \leq x \leq 5.99$.

x	0	1	2	3	4	5	6	7	8	9	exp
0.0	5.0000	4.9601	4.9202	4.8803	4.8405	4.8006	4.7608	4.7210	4.6812	4.6414	
0.1	4.6017	4.5620	4.5224	4.4828	4.4433	4.4038	4.3644	4.3251	4.2858	4.2465	
0.2	4.2074	4.1683	4.1294	4.0905	4.0517	4.0129	3.9743	3.9358	3.8974	3.8591	
0.3	3.8209	3.7828	3.7448	3.7070	3.6693	3.6317	3.5942	3.5569	3.5197	3.4827	
0.4	3.4458	3.4090	3.3724	3.3360	3.2997	3.2636	3.2276	3.1918	3.1561	3.1207	
0.5	3.0854	3.0503	3.0153	2.9806	2.9460	2.9116	2.8774	2.8434	2.8096	2.7760	−1
0.6	2.7425	2.7093	2.6763	2.6435	2.6109	2.5785	2.5463	2.5143	2.4825	2.4510	
0.7	2.4196	2.3885	2.3576	2.3270	2.2965	2.2663	2.2363	2.2065	2.1770	2.1476	
0.8	2.1186	2.0897	2.0611	2.0327	2.0045	1.9766	1.9489	1.9215	1.8943	1.8673	
0.9	1.8406	1.8141	1.7879	1.7619	1.7361	1.7106	1.6853	1.6602	1.6354	1.6109	
1.0	1.5866	1.5625	1.5386	1.5151	1.4917	1.4686	1.4457	1.4231	1.4007	1.3786	
1.1	1.3567	1.3350	1.3136	1.2924	1.2714	1.2507	1.2302	1.2100	1.1900	1.1702	
1.2	1.1507	1.1314	1.1123	1.0935	1.0749	1.0565	1.0383	1.0204	1.0027	9.8525	
1.3	9.6800	9.5098	9.3418	9.1759	9.0123	8.8508	8.6915	8.5343	8.3793	8.2264	
1.4	8.0757	7.9270	7.7804	7.6359	7.4934	7.3529	7.2145	7.0781	6.9437	6.8112	
1.5	6.6807	6.5522	6.4255	6.3008	6.1780	6.0571	5.9380	5.8208	5.7053	5.5917	
1.6	5.4799	5.3699	5.2616	5.1551	5.0503	4.9471	4.8457	4.7460	4.6479	4.5514	
1.7	4.4565	4.3633	4.2716	4.1815	4.0930	4.0059	3.9204	3.8364	3.7538	3.6727	−2
1.8	3.5930	3.5148	3.4380	3.3625	3.2884	3.2157	3.1443	3.0742	3.0054	2.9379	
1.9	2.8717	2.8067	2.7429	2.6803	2.6190	2.5588	2.4998	2.4419	2.3852	2.3295	
2.0	2.2750	2.2216	2.1692	2.1178	2.0675	2.0182	1.9699	1.9226	1.8763	1.8309	
2.1	1.7864	1.7429	1.7003	1.6586	1.6177	1.5778	1.5386	1.5003	1.4629	1.4262	
2.2	1.3903	1.3553	1.3209	1.2874	1.2545	1.2224	1.1911	1.1604	1.1304	1.1011	
2.3	1.0724	1.0444	1.0170	9.9031	9.6419	9.3867	9.1375	8.8940	8.6563	8.4242	
2.4	8.1975	7.9763	7.7603	7.5494	7.3436	7.1428	6.9469	6.7557	6.5691	6.3872	
2.5	6.2097	6.0366	5.8677	5.7031	5.5426	5.3861	5.2336	5.0849	4.9400	4.7988	
2.6	4.6612	4.5271	4.3965	4.2692	4.1453	4.0246	3.9070	3.7926	3.6811	3.5726	−3
2.7	3.4670	3.3642	3.2641	3.1667	3.0720	2.9798	2.8901	2.8028	2.7179	2.6354	
2.8	2.5551	2.4771	2.4012	2.3274	2.2557	2.1860	2.1182	2.0524	1.9884	1.9262	
2.9	1.8658	1.8071	1.7502	1.6948	1.6411	1.5889	1.5382	1.4890	1.4412	1.3949	
3.0	1.3499	1.3062	1.2639	1.2228	1.1829	1.1442	1.1067	1.0703	1.0350	1.0008	
3.1	9.6760	9.3544	9.0426	8.7403	8.4474	8.1635	7.8885	7.6219	7.3638	7.1136	
3.2	6.8714	6.6367	6.4095	6.1895	5.9765	5.7703	5.5706	5.3774	5.1904	5.0094	
3.3	4.8342	4.6648	4.5009	4.3423	4.1889	4.0406	3.8971	3.7584	3.6243	3.4946	−4
3.4	3.3693	3.2481	3.1311	3.0179	2.9086	2.8029	2.7009	2.6023	2.5071	2.4151	
3.5	2.3263	2.2405	2.1577	2.0778	2.0006	1.9262	1.8543	1.7849	1.7180	1.6534	
3.6	1.5911	1.5310	1.4730	1.4171	1.3632	1.3112	1.2611	1.2128	1.1662	1.1213	
3.7	1.0780	1.0363	9.9611	9.5740	9.2010	8.8417	8.4957	8.1624	7.8414	7.5324	
3.8	7.2348	6.9483	6.6726	6.4072	6.1517	5.9059	5.6694	5.4418	5.2228	5.0122	
3.9	4.8096	4.6148	4.4274	4.2473	4.0741	3.9076	3.7475	3.5936	3.4458	3.3037	−5
4.0	3.1671	3.0359	2.9099	2.7888	2.6726	2.5609	2.4536	2.3507	2.2518	2.1569	
4.1	2.0658	1.9783	1.8944	1.8138	1.7365	1.6624	1.5912	1.5230	1.4575	1.3948	
4.2	1.3346	1.2769	1.2215	1.1685	1.1176	1.0689	1.0221	9.7736	9.3447	8.9337	
4.3	8.5399	8.1627	7.8015	7.4555	7.1241	6.8069	6.5031	6.2123	5.9340	5.6675	
4.4	5.4125	5.1685	4.9350	4.7117	4.4979	4.2935	4.0980	3.9110	3.7322	3.5612	−6
4.5	3.3977	3.2414	3.0920	2.9492	2.8127	2.6823	2.5577	2.4386	2.3249	2.2162	
4.6	2.1125	2.0133	1.9187	1.8283	1.7420	1.6597	1.5810	1.5060	1.4344	1.3660	
4.7	1.3008	1.2386	1.1792	1.1226	1.0686	1.0171	9.6796	9.2113	8.7648	8.3391	
4.8	7.9333	7.5465	7.1779	6.8267	6.4920	6.1731	5.8693	5.5799	5.3043	5.0418	
4.9	4.7918	4.5538	4.3272	4.1115	3.9061	3.7107	3.5247	3.3476	3.1792	3.0190	−7
5.0	2.8665	2.7215	2.5836	2.4524	2.3277	2.2091	2.0963	1.9891	1.8872	1.7903	
5.1	1.6983	1.6108	1.5277	1.4487	1.3737	1.3024	1.2347	1.1705	1.1094	1.0515	
5.2	9.9644	9.4420	8.9462	8.4755	8.0288	7.6050	7.2028	6.8212	6.4592	6.1158	
5.3	5.7901	5.4813	5.1884	4.9106	4.6473	4.3977	4.1611	3.9368	3.7243	3.5229	−8
5.4	3.3320	3.1512	2.9800	2.8177	2.6640	2.5185	2.3807	2.2502	2.1266	2.0097	
5.5	1.8990	1.7942	1.6950	1.6012	1.5124	1.4283	1.3489	1.2737	1.2026	1.1353	
5.6	1.0718	1.0116	9.5479	9.0105	8.5025	8.0224	7.5686	7.1399	6.7347	6.3520	
5.7	5.9904	5.6488	5.3262	5.0215	4.7338	4.4622	4.2057	3.9636	3.7350	3.5193	−9
5.8	3.3157	3.1236	2.9424	2.7714	2.6100	2.4579	2.3143	2.1790	2.0513	1.9310	
5.9	1.8175	1.7105	1.6097	1.5147	1.4251	1.3407	1.2612	1.1863	1.1157	1.0492	

For $x > 0$, we have $(1 - x^{-2})\frac{1}{x\sqrt{2\pi}}e^{-x^2/2}dt < Q(x) < \frac{1}{x\sqrt{2\pi}}e^{-x^2/2}dt$. For large x we have $Q(x) \approx \frac{1}{x\sqrt{2\pi}}e^{-x^2/2}dt$.