

Exam in TSKS01 Digital Communication

- Exam code:** TEN1
- Date:** 2017-04-18 **Time:** 14:00–18:00
- Place:** KÅRA
- Teacher:** Emil Björnson, tel: 013 - 28 67 32
- Visiting exam:** Around 15 and 17
- Administrator:** Carina Lindström, 013 - 28 44 23, carina.e.lindstrom@liu.se
- Department:** ISY
- Allowed aids:** Pocket calculator with empty memory.
Olofsson: Tables and Formulas for Signal Theory.
- Number of tasks:** 7
- Solutions:** Will be published within five days after the exam at
<http://www.commsys.isy.liu.se/TSKS01>
- Result:** You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
- Exam return:** 2017-05-05, 12:45–13:00, Emil Björnson's office, Building B, top floor, corridor A between entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.
- Important:** **Solutions and answers must be given in English.**

Grading: This exam consists of three parts: an introductory task, a question part, and a problem part. The introductory task consists of two rather simple subtasks that test the ability to perform standard calculations. Each task in the question part and the problem part can give the number of points indicated in the margin. The question part can give you at most 10 points and the problem part can give you at most 20 points. For passing the exam, you need

- at least one of the two subtasks of the introductory task solved correctly,
- at least 3 points from the question part,
- at least 6 points from the problem part,
- and totally at least 14 points.

Grade limits:

- Grade three (ECTS C): 14 points,
- Grade four (ECTS B): 19 points,
- Grade five (ECTS A): 24 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

Introductory task

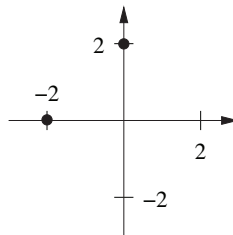
1 This task has to be solved correctly as partial fulfillment for passing the exam.

a. A binary linear block code has the following generator matrix:

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Give the corresponding parity-check matrix.

b. A binary modulation scheme uses the following two signal points.



Determine the error probability if we communicate over an AWGN channel where the noise has power spectral density $N_0 = 1$. The receiver uses an ML detector.

Question part

2 Explain how syndrome decoding works. Make sure to provide both a general description with arbitrary matrices (remember to define their dimensions!) and an example with numbers in the matrices. Explain why syndrome decoding leads to an ML detector. (5 p)

- 3 Are the following claims true or false? You do not need to explain your answers. (5 p)
- a. AWGN channels have a constant frequency response.
 - b. If all messages are equally likely, then ML detection cannot give a bit error probability that is larger than $1/2$.
 - c. Gray codes are used to reduce the bit error probability.
 - d. Repetition codes of even length are perfect codes.
 - e. The product of the parity-check matrix and the corresponding generator matrix gives the identity matrix.

For each of the claims above, a correct answer gives you +1 point, while an incorrect answer gives you -1 point. No answer give you 0 points for that claim, so a good strategy is to only give an answer if you are sure that it is correct. You cannot get less than 0 points totally from this task.

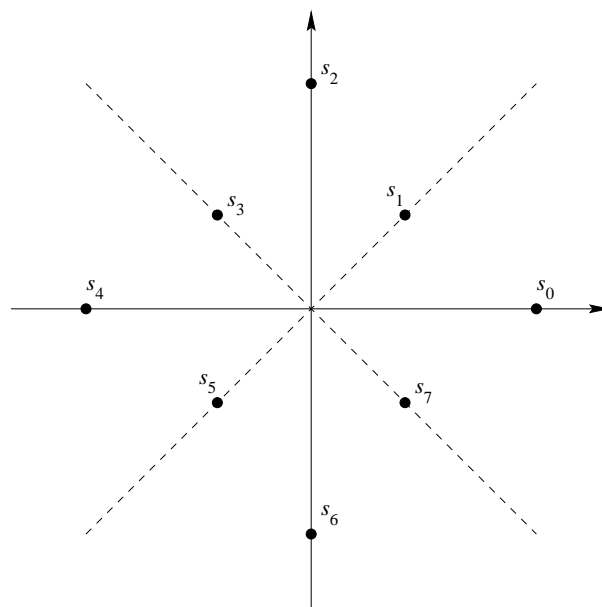
Problem part

- 4 A binary communication system uses the two equally probable signals $s_1(t)$ and $s_2(t)$ defined as (5 p)

$$s_1(t) = \begin{cases} A, & 0 < t < T_b, \\ 0, & \text{elsewhere,} \end{cases}$$
$$s_2(t) = \begin{cases} A, & 0 < t < aT_b, \\ 0, & \text{elsewhere,} \end{cases}$$

where $0 \leq a \leq 1$. The error probability of the system is supposed to be at most 10^{-5} . For what values of a is that possible if the signal-to-noise ratio E_b/N_0 is 16 dB? E_b is the average energy per bit. The noise is additive, white and Gaussian.

- 5 A digital communication system uses the signals s_i , $0 \leq i \leq 7$, in the constellation below for communication over an AWGN channel. The corresponding energies are E_i , $0 \leq i \leq 7$. The angles between the lines are all $\pi/4$ radians. (5 p)



How should the energies be chosen if the error probability is to be minimized, assuming equally probable signals and ML detection. The following demands have to be fulfilled:

- The signals are on the lines as indicated in the figure.
- $E_0 = E_2 = E_4 = E_6$ and $E_1 = E_3 = E_5 = E_7$.
- The average energy is constant.

SNR is assumed to be high. How good is the optimal solution compared to 8-PSK using the same average energy?

- 6 We want to send the message 011010 using the CRC code that is defined (5 p)
 by the polynomial $x^4 + x^3 + x + 1$.
- a. Draw a linear feedback shift register that generates this CRC code, by computing the remainder.
 - b. Compute the codeword that corresponds to this message.

7 A binary linear block code has the following generator matrix:

(5 p)

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

- a. Determine the length, dimension, size and minimum distance of the code.
- b. Decode the received vector (0111010).
- c. Prove whether or not this code satisfies the Hamming bound.

The Q -function, table of $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ for $0.00 \leq x \leq 5.99$.

x	0	1	2	3	4	5	6	7	8	9	exp
0.0	5.0000	4.9601	4.9202	4.8803	4.8405	4.8006	4.7608	4.7210	4.6812	4.6414	-1
0.1	4.6017	4.5620	4.5224	4.4828	4.4433	4.4038	4.3644	4.3251	4.2858	4.2465	
0.2	4.2074	4.1683	4.1294	4.0905	4.0517	4.0129	3.9743	3.9358	3.8974	3.8591	
0.3	3.8209	3.7828	3.7448	3.7070	3.6693	3.6317	3.5942	3.5569	3.5197	3.4827	
0.4	3.4458	3.4090	3.3724	3.3360	3.2997	3.2636	3.2276	3.1918	3.1561	3.1207	
0.5	3.0854	3.0503	3.0153	2.9806	2.9460	2.9116	2.8774	2.8434	2.8096	2.7760	
0.6	2.7425	2.7093	2.6763	2.6435	2.6109	2.5785	2.5463	2.5143	2.4825	2.4510	
0.7	2.4196	2.3885	2.3576	2.3270	2.2965	2.2663	2.2363	2.2065	2.1770	2.1476	
0.8	2.1186	2.0897	2.0611	2.0327	2.0045	1.9766	1.9489	1.9215	1.8943	1.8673	
0.9	1.8406	1.8141	1.7879	1.7619	1.7361	1.7106	1.6853	1.6602	1.6354	1.6109	
1.0	1.5866	1.5625	1.5386	1.5151	1.4917	1.4686	1.4457	1.4231	1.4007	1.3786	
1.1	1.3567	1.3350	1.3136	1.2924	1.2714	1.2507	1.2302	1.2100	1.1900	1.1702	
1.2	1.1507	1.1314	1.1123	1.0935	1.0749	1.0565	1.0383	1.0204	1.0027	9.8525	
1.3	9.6800	9.5098	9.3418	9.1759	9.0123	8.8508	8.6915	8.5343	8.3793	8.2264	
1.4	8.0757	7.9270	7.7804	7.6359	7.4934	7.3529	7.2145	7.0781	6.9437	6.8112	
1.5	6.6807	6.5522	6.4255	6.3008	6.1780	6.0571	5.9380	5.8208	5.7053	5.5917	
1.6	5.4799	5.3699	5.2616	5.1551	5.0503	4.9471	4.8457	4.7460	4.6479	4.5514	
1.7	4.4565	4.3633	4.2716	4.1815	4.0930	4.0059	3.9204	3.8364	3.7538	3.6727	
1.8	3.5930	3.5148	3.4380	3.3625	3.2884	3.2157	3.1443	3.0742	3.0054	2.9379	
1.9	2.8717	2.8067	2.7429	2.6803	2.6190	2.5588	2.4998	2.4419	2.3852	2.3295	
2.0	2.2750	2.2216	2.1692	2.1178	2.0675	2.0182	1.9699	1.9226	1.8763	1.8309	
2.1	1.7864	1.7429	1.7003	1.6586	1.6177	1.5778	1.5386	1.5003	1.4629	1.4262	
2.2	1.3903	1.3553	1.3209	1.2874	1.2545	1.2224	1.1911	1.1604	1.1304	1.1011	
2.3	1.0724	1.0444	1.0170	9.9031	9.6419	9.3867	9.1375	8.8940	8.6563	8.4242	
2.4	8.1975	7.9763	7.7603	7.5494	7.3436	7.1428	6.9469	6.7557	6.5691	6.3872	
2.5	6.2097	6.0366	5.8677	5.7031	5.5426	5.3861	5.2336	5.0849	4.9400	4.7988	
2.6	4.6612	4.5271	4.3965	4.2692	4.1453	4.0246	3.9070	3.7926	3.6811	3.5726	
2.7	3.4670	3.3642	3.2641	3.1667	3.0720	2.9798	2.8901	2.8028	2.7179	2.6354	
2.8	2.5551	2.4771	2.4012	2.3274	2.2557	2.1860	2.1182	2.0524	1.9884	1.9262	
2.9	1.8658	1.8071	1.7502	1.6948	1.6411	1.5889	1.5382	1.4890	1.4412	1.3949	
3.0	1.3499	1.3062	1.2639	1.2228	1.1829	1.1442	1.1067	1.0703	1.0350	1.0008	
3.1	9.6760	9.3544	9.0426	8.7403	8.4474	8.1635	7.8885	7.6219	7.3638	7.1136	
3.2	6.8714	6.6367	6.4095	6.1895	5.9765	5.7703	5.5706	5.3774	5.1904	5.0094	
3.3	4.8342	4.6648	4.5009	4.3423	4.1889	4.0406	3.8971	3.7584	3.6243	3.4946	
3.4	3.3693	3.2481	3.1311	3.0179	2.9086	2.8029	2.7009	2.6023	2.5071	2.4151	
3.5	2.3263	2.2405	2.1577	2.0778	2.0006	1.9262	1.8543	1.7849	1.7180	1.6534	
3.6	1.5911	1.5310	1.4730	1.4171	1.3632	1.3112	1.2611	1.2128	1.1662	1.1213	
3.7	1.0780	1.0363	9.9611	9.5740	9.2010	8.8417	8.4957	8.1624	7.8414	7.5324	
3.8	7.2348	6.9483	6.6726	6.4072	6.1517	5.9059	5.6694	5.4418	5.2228	5.0122	
3.9	4.8096	4.6148	4.4274	4.2473	4.0741	3.9076	3.7475	3.5936	3.4458	3.3037	
4.0	3.1671	3.0359	2.9099	2.7888	2.6726	2.5609	2.4536	2.3507	2.2518	2.1569	
4.1	2.0658	1.9783	1.8944	1.8138	1.7365	1.6624	1.5912	1.5230	1.4575	1.3948	
4.2	1.3346	1.2769	1.2215	1.1685	1.1176	1.0689	1.0221	9.7736	9.3447	8.9337	
4.3	8.5399	8.1627	7.8015	7.4555	7.1241	6.8069	6.5031	6.2123	5.9340	5.6675	
4.4	5.4125	5.1685	4.9350	4.7117	4.4979	4.2935	4.0980	3.9110	3.7322	3.5612	
4.5	3.3977	3.2414	3.0920	2.9492	2.8127	2.6823	2.5577	2.4386	2.3249	2.2162	
4.6	2.1125	2.0133	1.9187	1.8283	1.7420	1.6597	1.5810	1.5060	1.4344	1.3660	
4.7	1.3008	1.2386	1.1792	1.1226	1.0686	1.0171	9.6796	9.2113	8.7648	8.3391	
4.8	7.9333	7.5465	7.1779	6.8267	6.4920	6.1731	5.8693	5.5799	5.3043	5.0418	
4.9	4.7918	4.5538	4.3272	4.1115	3.9061	3.7107	3.5247	3.3476	3.1792	3.0190	
5.0	2.8665	2.7215	2.5836	2.4524	2.3277	2.2091	2.0963	1.9891	1.8872	1.7903	
5.1	1.6983	1.6108	1.5277	1.4487	1.3737	1.3024	1.2347	1.1705	1.1094	1.0515	
5.2	9.9644	9.4420	8.9462	8.4755	8.0288	7.6050	7.2028	6.8212	6.4592	6.1158	
5.3	5.7901	5.4813	5.1884	4.9106	4.6473	4.3977	4.1611	3.9368	3.7243	3.5229	
5.4	3.3320	3.1512	2.9800	2.8177	2.6640	2.5185	2.3807	2.2502	2.1266	2.0097	
5.5	1.8990	1.7942	1.6950	1.6012	1.5124	1.4283	1.3489	1.2737	1.2026	1.1353	
5.6	1.0718	1.0116	9.5479	9.0105	8.5025	8.0224	7.5686	7.1399	6.7347	6.3520	
5.7	5.9904	5.6488	5.3262	5.0215	4.7338	4.4622	4.2057	3.9636	3.7350	3.5193	
5.8	3.3157	3.1236	2.9424	2.7714	2.6100	2.4579	2.3143	2.1790	2.0513	1.9310	
5.9	1.8175	1.7105	1.6097	1.5147	1.4251	1.3407	1.2612	1.1863	1.1157	1.0492	

For $x > 0$, we have $(1 - x^{-2}) \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} < Q(x) < \frac{1}{x\sqrt{2\pi}} e^{-x^2/2}$. For large x we have $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2}$.