Exam in TSKS01 Digital Communication

Exam code:	TEN1
Date:	2017-01-09 **Time:** 14:00–18:00
Place:	U6 (20 seats), U7 (18 seats)
Teacher:	Emil Björnson, tel: 013 - 28 67 32
Visiting exam:	Around 15 and 17
Administrator:	Carina Lindström, 013 - 28 44 23, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Pocket calculator with empty memory. Olofsson: Tables and Formulas for Signal Theory.
Number of tasks:	7
Solutions:	Will be published within five days after the exam at http://www.commsys.isy.liu.se/TSKS01
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	2017-01-30, 12:30–12.50, Emil Björnson's office, Building B, top floor, corridor A between entrances 27–29. After that in the student office of Dept. of EE. (ISY), Building B, Corridor D, between Entrances 27–29, right next to Café Java.

Important: Solutions and answers must be given in English.

Grading: This exam consists of three parts: an introductory task, a question part, and a problem part. The introductory task consists of two rather simple subtasks that test the ability to perform standard calculations. Each task in the question part and the problem part can give the number of points indicated in the margin. The question part can give you at most 10 points and the problem part can give you at most 20 points. For passing the exam, you need

- at least one of the two subtasks of the introductory task solved correctly,
- at least 3 points from the question part,
- at least 6 points from the problem part,
- and totally at least 14 points.

Grade limits:

- Grade three (ECTS C): 14 points,
- Grade four (ECTS B): 19 points,
- Grade five (ECTS A): 24 points.

Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

Introductory task

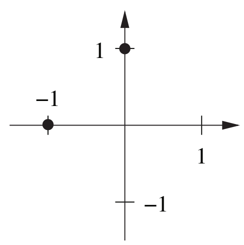
1 This task has to be solved correctly as partial fulfillment for passing the exam.

a. A binary linear block code has the following generator matrix:

$$G = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

Determine all the codewords.

b. A binary modulation scheme uses the following two signal points.



Determine the error probability if we communicate over an AWGN channel where the noise has power spectral density $N_0/2 = 0.5$. The receiver uses an ML detector.

Question part

2 Explain how cyclic redundancy check (CRC) codes work. Make sure to describe and exemplify how to generate codewords and how to detect errors. Give an example of a linear feedback shift register implementation of a CRC decoding. (5 p)

3 Are the following claims true or false? You do not need to explain your answers. (5 p)

a. For a given symbol time, a raised cosine pulse requires more bandwidth than a sinc pulse.

b. A phase-locked loop is used to decode cyclic codes.

c. The rate of an error control code can be between -1 and $+1$.

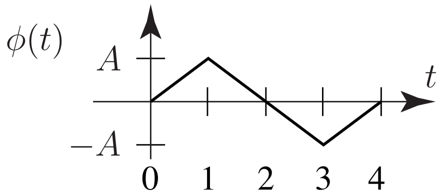
d. The union bound is close to the exact error probability at high SNR.

e. The signal space diagram is identical for QPSK and 4-QAM.

For each of the claims above, a correct answer gives you $+1$ point, while an incorrect answer gives you -1 point. No answer give you 0 points for that claim, so a good strategy is to only give an answer if you are sure that it is correct. You cannot get less than 0 points totally from this task.

Problem part

4 We communicate over an AWGN channel using BPSK with $s_0(t) = \phi(t)$ and $s_1(t) = -\phi(t)$. The pulse function $\phi(t)$ is defined as: (5 p)



The noise has power spectral density $N_0/2$. Answer the following questions:

a. What is the exact symbol error probability, using ML detection? Write the expression as a function of A and N_0. (2p)

b. We want to achieve an error probability of 10^{-4}. How large must A be to achieve this, assuming $N_0 = 1$? (2p)

c. Propose an alternative pulse function with the same energy as $\phi(t)$, but smaller peak values (the largest value is smaller than A and the smallest value is larger than $-A$). (1p)

5 Show that a linear block code must have the following properties: (5 p)

a. The all-zero vector is in the code.

b. The minimum distance is equal to the minimum weight among the non-zero codewords.

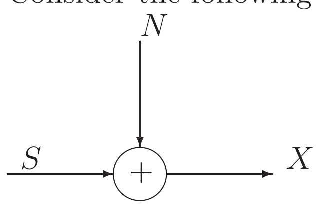
6 A BPSK constellation is used to communicate over a two-tap channel, where the information symbols are either $s[n] = -1$ or $s[n] = +1$. The received signal is (5 p)

$$Z[k] = 2s[k] - s[k-1] + W_\gamma[k],$$

where $W_\gamma[k]$ is independent Gaussian noise with zero mean and variance $N_0/2$. We assume that $s[-1] = +1$. Find the ML estimate of the sequence $(s[0], s[1], s[2])$, given the received signals $z[0] = -0.5$, $z[1] = 2$, and $z[2] = 1$.

Alternative problem, only for previous-year students (before the course contained dispersive channels):

In this problem, we consider MAP detection. Consider the following transmission system:

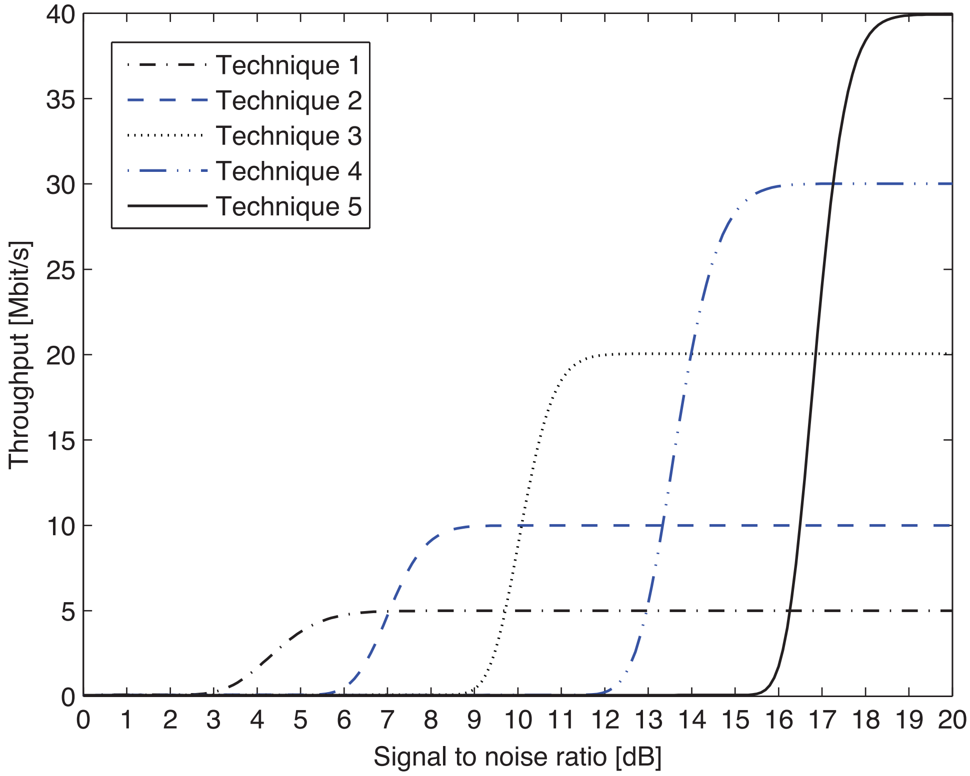


The input variable S assumes the value $\sqrt{E}$ with probability p and the value $-\sqrt{E}$ with probability $1-p$. The sent message is disturbed by a Gaussian variable N, with mean 0 and variance $N_0/2$, independent of S.

a. Determine the optimum decision rule such that the resulting error probability is minimized. (4p)

b. Determine the error probability for the given situation assuming your decision rule. (1p)

7 The throughput (bit/s) in packet data transmission with five different combinations of modulation and coding are shown in the following figure: (5 p)



Assume that all five combinations use the same modulation scheme, but different error control coding. The symbol time is $T = 10^{-7}$ s. Answer the following questions (and motivate your answers):

a. Technique 5 is achieved by uncoded transmission. Which modulation scheme could have been used and why? (2p)

b. One of the techniques uses a binary linear code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Which technique is this? (1p)

c. Technique 1 is achieved by a repetition code. Provide a generator matrix for this code. (2p)

The Q-function, table of $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ for $0.00 \le x \le 5.99$.

x	0	1	2	3	4	5	6	7	8	9	exp
0.0	5.0000	4.9601	4.9202	4.8803	4.8405	4.8006	4.7608	4.7210	4.6812	4.6414	
0.1	4.6017	4.5620	4.5224	4.4828	4.4433	4.4038	4.3644	4.3251	4.2858	4.2465	
0.2	4.2074	4.1683	4.1294	4.0905	4.0517	4.0129	3.9743	3.9358	3.8974	3.8591	
0.3	3.8209	3.7828	3.7448	3.7070	3.6693	3.6317	3.5942	3.5569	3.5197	3.4827	
0.4	3.4458	3.4090	3.3724	3.3360	3.2997	3.2636	3.2276	3.1918	3.1561	3.1207	
0.5	3.0854	3.0503	3.0153	2.9806	2.9460	2.9116	2.8774	2.8434	2.8096	2.7760	−1
0.6	2.7425	2.7093	2.6763	2.6435	2.6109	2.5785	2.5463	2.5143	2.4825	2.4510	
0.7	2.4196	2.3885	2.3576	2.3270	2.2965	2.2663	2.2363	2.2065	2.1770	2.1476	
0.8	2.1186	2.0897	2.0611	2.0327	2.0045	1.9766	1.9489	1.9215	1.8943	1.8673	
0.9	1.8406	1.8141	1.7879	1.7619	1.7361	1.7106	1.6853	1.6602	1.6354	1.6109	
1.0	1.5866	1.5625	1.5386	1.5151	1.4917	1.4686	1.4457	1.4231	1.4007	1.3786	
1.1	1.3567	1.3350	1.3136	1.2924	1.2714	1.2507	1.2302	1.2100	1.1900	1.1702	
1.2	1.1507	1.1314	1.1123	1.0935	1.0749	1.0565	1.0383	1.0204	1.0027	9.8525	
1.3	9.6800	9.5098	9.3418	9.1759	9.0123	8.8508	8.6915	8.5343	8.3793	8.2264	
1.4	8.0757	7.9270	7.7804	7.6359	7.4934	7.3529	7.2145	7.0781	6.9437	6.8112	
1.5	6.6807	6.5522	6.4255	6.3008	6.1780	6.0571	5.9380	5.8208	5.7053	5.5917	
1.6	5.4799	5.3699	5.2616	5.1551	5.0503	4.9471	4.8457	4.7460	4.6479	4.5514	
1.7	4.4565	4.3633	4.2716	4.1815	4.0930	4.0059	3.9204	3.8364	3.7538	3.6727	−2
1.8	3.5930	3.5148	3.4380	3.3625	3.2884	3.2157	3.1443	3.0742	3.0054	2.9379	
1.9	2.8717	2.8067	2.7429	2.6803	2.6190	2.5588	2.4998	2.4419	2.3852	2.3295	
2.0	2.2750	2.2216	2.1692	2.1178	2.0675	2.0182	1.9699	1.9226	1.8763	1.8309	
2.1	1.7864	1.7429	1.7003	1.6586	1.6177	1.5778	1.5386	1.5003	1.4629	1.4262	
2.2	1.3903	1.3553	1.3209	1.2874	1.2545	1.2224	1.1911	1.1604	1.1304	1.1011	
2.3	1.0724	1.0444	1.0170	9.9031	9.6419	9.3867	9.1375	8.8940	8.6563	8.4242	
2.4	8.1975	7.9763	7.7603	7.5494	7.3436	7.1428	6.9469	6.7557	6.5691	6.3872	
2.5	6.2097	6.0366	5.8677	5.7031	5.5426	5.3861	5.2336	5.0849	4.9400	4.7988	
2.6	4.6612	4.5271	4.3965	4.2692	4.1453	4.0246	3.9070	3.7926	3.6811	3.5726	−3
2.7	3.4670	3.3642	3.2641	3.1667	3.0720	2.9798	2.8901	2.8028	2.7179	2.6354	
2.8	2.5551	2.4771	2.4012	2.3274	2.2557	2.1860	2.1182	2.0524	1.9884	1.9262	
2.9	1.8658	1.8071	1.7502	1.6948	1.6411	1.5889	1.5382	1.4890	1.4412	1.3949	
3.0	1.3499	1.3062	1.2639	1.2228	1.1829	1.1442	1.1067	1.0703	1.0350	1.0008	
3.1	9.6760	9.3544	9.0426	8.7403	8.4474	8.1635	7.8885	7.6219	7.3638	7.1136	
3.2	6.8714	6.6367	6.4095	6.1895	5.9765	5.7703	5.5706	5.3774	5.1904	5.0094	
3.3	4.8342	4.6648	4.5009	4.3423	4.1889	4.0406	3.8971	3.7584	3.6243	3.4946	−4
3.4	3.3693	3.2481	3.1311	3.0179	2.9086	2.8029	2.7009	2.6023	2.5071	2.4151	
3.5	2.3263	2.2405	2.1577	2.0778	2.0006	1.9262	1.8543	1.7849	1.7180	1.6534	
3.6	1.5911	1.5310	1.4730	1.4171	1.3632	1.3112	1.2611	1.2128	1.1662	1.1213	
3.7	1.0780	1.0363	9.9611	9.5740	9.2010	8.8417	8.4957	8.1624	7.8414	7.5324	
3.8	7.2348	6.9483	6.6726	6.4072	6.1517	5.9059	5.6694	5.4418	5.2228	5.0122	
3.9	4.8096	4.6148	4.4274	4.2473	4.0741	3.9076	3.7475	3.5936	3.4458	3.3037	−5
4.0	3.1671	3.0359	2.9099	2.7888	2.6726	2.5609	2.4536	2.3507	2.2518	2.1569	
4.1	2.0658	1.9783	1.8944	1.8138	1.7365	1.6624	1.5912	1.5230	1.4575	1.3948	
4.2	1.3346	1.2769	1.2215	1.1685	1.1176	1.0689	1.0221	9.7736	9.3447	8.9337	
4.3	8.5399	8.1627	7.8015	7.4555	7.1241	6.8069	6.5031	6.2123	5.9340	5.6675	
4.4	5.4125	5.1685	4.9350	4.7117	4.4979	4.2935	4.0980	3.9110	3.7322	3.5612	−6
4.5	3.3977	3.2414	3.0920	2.9492	2.8127	2.6823	2.5577	2.4386	2.3249	2.2162	
4.6	2.1125	2.0133	1.9187	1.8283	1.7420	1.6597	1.5810	1.5060	1.4344	1.3660	
4.7	1.3008	1.2386	1.1792	1.1226	1.0686	1.0171	9.6796	9.2113	8.7648	8.3391	
4.8	7.9333	7.5465	7.1779	6.8267	6.4920	6.1731	5.8693	5.5799	5.3043	5.0418	
4.9	4.7918	4.5538	4.3272	4.1115	3.9061	3.7107	3.5247	3.3476	3.1792	3.0190	−7
5.0	2.8665	2.7215	2.5836	2.4524	2.3277	2.2091	2.0963	1.9891	1.8872	1.7903	
5.1	1.6983	1.6108	1.5277	1.4487	1.3737	1.3024	1.2347	1.1705	1.1094	1.0515	
5.2	9.9644	9.4420	8.9462	8.4755	8.0288	7.6050	7.2028	6.8212	6.4592	6.1158	
5.3	5.7901	5.4813	5.1884	4.9106	4.6473	4.3977	4.1611	3.9368	3.7243	3.5229	−8
5.4	3.3320	3.1512	2.9800	2.8177	2.6640	2.5185	2.3807	2.2502	2.1266	2.0097	
5.5	1.8990	1.7942	1.6950	1.6012	1.5124	1.4283	1.3489	1.2737	1.2026	1.1353	
5.6	1.0718	1.0116	9.5479	9.0105	8.5025	8.0224	7.5686	7.1399	6.7347	6.3520	
5.7	5.9904	5.6488	5.3262	5.0215	4.7338	4.4622	4.2057	3.9636	3.7350	3.5193	−9
5.8	3.3157	3.1236	2.9424	2.7714	2.6100	2.4579	2.3143	2.1790	2.0513	1.9310	
5.9	1.8175	1.7105	1.6097	1.5147	1.4251	1.3407	1.2612	1.1863	1.1157	1.0492	

For $x > 0$, we have $(1 - x^{-2}) \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt < Q(x) < \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$. For large x we have $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} dt$.