TSDT74 Radio Communication

Solutions to the exam 2015-06-01

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PART A

A.1 The noise figure F is defined as the ratio $F = T/T_0$, where T is the system's noise temperature and $T_o = 290K$ is a reference temperature.

A.2 The coherence time represents how fast the channel characteristics are changing with time. For a time period of at most the coherence time the channel can be considered as time-invariant.

A.3

- **Half-power bandwidth:** This is the interval between the two frequencies at which the power spectrum is 3 dB below its peak value.
- **Equivalent noise bandwidth:** This measures the basis of a rectangle whose height is $max_f \Phi_s(f)$ and whose area is one-half of the power of the modulated signal.
- **Null-to-null bandwidth:** This represents the width of the main spectral lobe.

A.4 The length of the cyclic prefix for the OFDM symbol is $G = T_G * f_s = 0.15 * 10^{-3} * 18 * 10^4 = 27$ samples.

A.5 The coherence bandwidth B_m is approximately inversely proportional to the multipath (delay) spread, i.e.

$$B_m \approx 1/T_m = 1/(20 * 10^{-6}) = 50 \ \text{ kHz}.$$

A.6 The efficiency of a selective-repeat ARQ scheme is given by

$$\eta = \frac{k}{n} P_a = \frac{7}{8} 0.8 = 0.7 = 70\%$$

The expected message delay is

$$D = \frac{1}{P_a} \frac{n}{r_c} + t_r = \frac{t_p}{P_a} + t_r = \frac{50}{0.8} + 15 = 77.5 \text{ ms}.$$

A.7 The maximal-ratio combining performs better in comparison with the selection diversity combining. If we neglect the information about the channel gain and only use the phase shift information we can reduce the complexity of the maximal-ratio combining method. In this way we obtain the equal-gain combining method.

A.8 In the case of block interleaving we have two equivalent memory blocks at the transmitter and the receiver. In the case of convolutional interleaving we have one common memory block which consists of shift registers of different length. The contents of the registers are consecutively shifted over the channel. The convolutional interleaver has shorter delay.

A.9 The fast fading is caused by the reflections of the EM-waves by the local objects to the receiver. Due to this great changes in the signal quality are observed even when the receiver slightly changes its position.

A.10

The frequency hop spread spectrum signal is constructed by modulating the original signal on a sequence of carrier frequencies $\{f_1, f_2, \ldots, f_N\}$ that are chosen in a seemingle random way (most often by using PN-sequences). A new carrire frequency is chosen for each "chip"-interval. The carrier frequency, f_i, is constant for a hopping interval and then changes to another (preselected) frequency for the next interval of time.

PART B

B.1 In an ALOHA communication system, the transmitted packets are passing Rayleigh fading fading channel. We assuming that the packet transmissions are independent and that the average received power is the same for each transmission.

a) Let us assume that these 2 colliding packets have been received with powers P_1 and P_2, respectively. The probability for a resolved collision is then:

$$\begin{aligned} P_{\text{cap}} &= Pr\{\frac{P_1}{P_2} \geq 10^{1.3}\} + Pr\{\frac{P_2}{P_1} \geq 10^{1.3}\} \\ &\approx 2 \times Pr\{\frac{P_2}{P_1} \geq 20\}. \end{aligned}$$

Since P_1 and P_2 are independent and exponentially distributed, we can calculate the required probability in the following way:

$$\begin{aligned} P_{\text{cap}} &\approx 2\int_0^\infty \left(\int_{20\gamma_1}^\infty \frac{1}{\gamma_0^2} e^{-\frac{\gamma_1}{\gamma_0}} e^{-\frac{\gamma_2}{\gamma_0}} d\gamma_2\right) d\gamma_1 \\ &= \frac{2}{\gamma_0^2}\int_0^\infty e^{-\frac{\gamma_1}{\gamma_0}} \left(\int_{20\gamma_1}^\infty e^{-\frac{\gamma_2}{\gamma_0}} d\gamma_2\right) d\gamma_1 \\ &= \frac{2}{\gamma_0}\int_0^\infty e^{-\frac{\gamma_1}{\gamma_0}} \left[e^{-\frac{\gamma_2}{\gamma_0}}\right]_{20\gamma_1}^\infty d\gamma_1 \\ &= \frac{2}{\gamma_0}\int_0^\infty e^{-\frac{\gamma_1}{\gamma_0}} e^{-\frac{20\gamma_1}{\gamma_0}} d\gamma_1 \\ &= \frac{2}{\gamma_0}\int_0^\infty e^{-\frac{21\gamma_1}{\gamma_0}} d\gamma_1 \\ &= \frac{2}{21}\left[e^{-\frac{21\gamma_1}{\gamma_0}}\right]_0^\infty d\gamma_1 \\ &= \frac{2}{21}. \end{aligned}$$

b) When the mean-SNR is 23 dB we have

$$\gamma_0 = 10^{2.3} \approx 200$$

and

$$\begin{aligned} P_{\text{PL}} &= Pr\{\Gamma \leq 10^{1.3}\} \approx Pr\{\Gamma \leq 20\} \\ &= \int_0^{20} \frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} d\gamma \\ &= 1 - e^{-20/200} = 1 - e^{-0.1} \approx 0.095. \end{aligned}$$

c) The throughput λ of this ALOHA system as a functiuon of the attempt rate η can be calculated as:

$$\begin{aligned} \lambda &= \eta Pr\{\text{successfultransmission}\} \\ &= \eta(Pr\{\text{succ.trans.singlepacket}\} \\ &\quad + Pr\{\text{succ.trans.doublepackets}\}) \\ &= \eta\left((1 - P_{\text{PL}}) * e^{-\eta} + P_{\text{cap}} * \eta * e^{-\eta}\right) \\ &= \eta e^{-\eta}(0.905 + \frac{2}{21}\eta) \end{aligned}$$

In order to find the optimum value of λ we calculate the derivative:

$$\begin{aligned} \frac{d\lambda}{d\eta} &= e^{-\eta}(0.905 + \frac{2}{21}\eta) - \eta e^{-\eta}(0.905 + \frac{2}{21}\eta) \\ &\quad + \eta e^{-\eta}\frac{2}{21} \\ &= e^{-\eta}(0.905 + \frac{2}{21}\eta - 0.905\eta - \frac{2}{21}\eta^2 + \frac{2}{21}\eta \\ &\approx e^{-\eta}(-0.095\eta^2 - 0.715\eta + 0.905) \end{aligned}$$

The positive root of the quadratic equation is approximately 1.103 which provides maximum throughput $\lambda_{max} = \lambda(1.1) \approx 0.37$.

B.2

a) The average BER for coherent BFSK over flat Rayleigh fading channel according to equation (4.139) in the book is

$$\begin{aligned} P_b &= \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{2 + \gamma_0}}\right) \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{10}{12}}\right) \approx 0.043 \end{aligned}$$

b) When introducing parity-check bits the average SNR is decreased by a fraction of $k/n = 11/15 \approx 0.733$ which results in a new average SNR per coded bit $\gamma_1 = 11\gamma_0/15 \approx 7.33$. The coded BER is then

$$\begin{aligned} P_{cb} &= \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_1}{2 + \gamma_1}}\right) \\ &\approx \frac{1}{2}\left(1 - \sqrt{\frac{7.33}{9.33}}\right) \approx 0.0568 \end{aligned}$$

The error correcting capability of the code is $\nu = \lfloor(d-1)/2\rfloor = 1$. The codeword error probability can be computed as

$$\begin{aligned} P_{cw} &= 1 - \sum_{i=0}^{\nu} \binom{n}{i} P_{cb}^i (1 - P_{cb})^{n-i} \\ &= 1 - (1 - P_{cb})^{15} - 15P_{cb}(1 - P_{cb})^{14} \approx 0.2083 \end{aligned}$$

This leads to an average BER of

$$P_b \approx \frac{d}{n} P_{cw} \approx 0.04166$$

B.3 The parameters of the communication link are:

- frequency: $f = 10^9$ Hz;
- wavelength: $\lambda = c/f = 0.3$ m;
- transmit antenna gain : $[G_T]_{dB} = 4$ dBd $= 6.1$ dB;
- transmit antenna gain height: h_T m $= [h_T]_{dB}$ dB m ;
- transmitter's feeder loss (non-existent): $[L_{TF}]_{dB} = 0$ dB;
- receive antenna gain : $[G_R]_{dB} = 2.5$ dB;
- receive antenna gain height: $h_R = 10$ m;
- The distance: $R = 10$ km $= 10^4$ m;
- receiver's feeder loss: $[L_{RF}]_{dB} = \frac{2.5}{100} 20$ dB $= 0.5$ dB;
- receiver's noise figure: $F_R = 20$ dB;
- transmission bandwidth: $B = 50$ kHz $\Rightarrow [B]_{dB} \approx 47$ dB;
- required SNR: $[S/N]_{dB} \geq 30$ dB;
- Boltzmann's constant $k = 1.38 \cdot 10^{-23}$ J/K $\Rightarrow [k]_{dB} \approx -228.6$ dB;
- transmit power: P_T W $= [P_T]_{dB}$ dBW;

From these parameters we need to calculate:

- the free space loss (according to the PE-model): $[L_{PE}]_{dB} = \left[\frac{R^4}{h_T^2 h_R^2} \right]_{dB} = 140 - 2[h_T]_{dB}$;
- receiver's noise temperature: $[T_R]_{dB} = [F_R T_0]_{dB} = [290 F_R]_{dB} = 44.624$ dB;

The link-budget equation is taking the form:

$$\begin{aligned} [S/N]_{dB} &= [P_T]_{dB} + [G_T]_{dB} + [G_R]_{dB} - [T_R]_{dB} \\ &\quad -[L_{PE}]_{dB} - [L_{TF}]_{dB} - [L_{RF}]_{dB} \\ &\quad -[B]_{dB} - [k]_{dB} \\ &= 5.076 + [P_T]_{dB} + 2[h_T]_{dB} \geq 30 \end{aligned}$$

which results in the inequality

$$[P_T]_{dB} + 2[h_T]_{dB} \geq 24.924$$

a) In this case we have $[h_T]_{dB} = 10 * \log_{10}(25) \approx 13.4$ and thus

$$[P_T]_{dB} \geq 24.924 - 2[h_T]_{dB} \geq -3.035 \text{ dBW}$$

which is

$$P_T \geq 10^{-0.3035} \approx 0.5 \text{ W}$$

b) In this case we have $[P_T]_{dB} = 10 * \log_{10}(10) = 10$ dBW and thus

$$[h_T]_{dB} \geq (24.924 - [P_T]_{dB})/2 \geq 7.462$$

which is $h_T \geq 10^{0.7412}$ m ≈ 5.57 m.

B.4 According to equations (5.52) and (5.53), the probability density function of the received SNR after switched combining with threshold γ_x is given by

$$p_\Gamma(\gamma) = \begin{cases} (1 + q_x)\frac{1}{\gamma_0} e^{-\gamma/\gamma_0}, & \text{if} \quad \gamma \geq \gamma_x, \\ q_x \frac{1}{\gamma_0} e^{-\gamma/\gamma_0}, & \text{if} \quad 0 \leq \gamma < \gamma_x, \end{cases}$$

where $q_x = 1 - e^{-\gamma_x/\gamma_0}$. In our case we have $\gamma_0 = \gamma_x = 10$ dB $= 10$ and thus $q_x = 1 - e^{-\gamma_x/\gamma_0} = 1 - e^{-1}$. The BER for the binary DPSK modulation over AWGN channel with SNR γ is given by

$$P_b(\gamma) = \frac{1}{2} e^{-\gamma}.$$

a) The average BER after the switched combining is derived as

$$\begin{aligned} P_b &= E[P_b(\gamma)] = \int_0^\infty \tfrac{1}{2} e^{-\gamma} p_\Gamma(\gamma) d\gamma \\ &= \int_0^{\gamma_x} \tfrac{1}{2} e^{-\gamma} q_x \tfrac{1}{\gamma_0} e^{-\gamma/\gamma_0} d\gamma \\ &\quad + \int_{\gamma_x}^\infty \tfrac{1}{2} e^{-\gamma} (1 + q_x) \tfrac{1}{\gamma_0} e^{-\gamma/\gamma_0} d\gamma \\ &= \tfrac{q_x}{2\gamma_0} \int_0^\infty \gamma e^{-\gamma(\gamma_0+1)/(\gamma_0)} d\gamma \\ &\quad + \tfrac{1}{2\gamma_0} \int_{\gamma_x}^\infty \gamma e^{-\gamma(\gamma_0+1)/(\gamma_0)} d\gamma \\ &= \tfrac{q_x}{2\gamma_0} \tfrac{-\gamma_0}{\gamma_0+1} \; e^{-\gamma(\gamma_0+1)/(\gamma_0)} \Big|_0^\infty \\ &\quad + \tfrac{1}{2\gamma_0} \tfrac{-\gamma_0}{\gamma_0+1} \; e^{-\gamma(\gamma_0+1)/(\gamma_0)} \Big|_{\gamma_x}^\infty \\ &= \tfrac{q_x}{2(\gamma_0+1)} + \tfrac{e^{-\gamma_x(\gamma_0+1)/(\gamma_0)}}{2(\gamma_0+1)} \\ &= \tfrac{1-e^{-1}+e^{-11}}{22} \approx 0.028732 \approx 2.9\%. \end{aligned}$$

b) According to equation (5.23) the probability density function for the SNR for MRC with two independent Rayleigh fading branches is

$$p_\Gamma(\gamma) = \frac{\gamma}{\gamma_0^2} e^{-\gamma/\gamma_0} \text{if } \gamma \geq 0$$

and 0 otherwise. The average BER when MRC can now be calculated as

$$
\begin{aligned}
P_b &= E[P_b(\gamma)] = \int_0^\infty \tfrac{1}{2}e^{-\gamma} p_\Gamma(\gamma)d\gamma \\
&= \int_0^\infty \tfrac{1}{2}e^{-\gamma} \tfrac{\gamma}{\gamma_0^2} e^{-\gamma/\gamma_0} d\gamma \\
&= \tfrac{1}{2\gamma_0^2} \int_0^\infty \gamma e^{-\gamma(\gamma_0+1)/(\gamma_0)} d\gamma \\
&= -\tfrac{1}{2\gamma_0(\gamma_0+1)} \gamma e^{-\gamma(\gamma_0+1)/(\gamma_0)} \Big|_0^\infty \\
&\quad \tfrac{1}{2\gamma_0(\gamma_0+1)} \int_0^\infty e^{-\gamma(\gamma_0+1)/(\gamma_0)} d\gamma \\
&= -\tfrac{1}{2(\gamma_0+1)^2} e^{-\gamma(\gamma_0+1)/(\gamma_0)} \Big|_0^\infty \\
&= \tfrac{1}{2(\gamma_0+1)^2} = \tfrac{1}{242} \approx 0.0041322 \approx 0.4\%.
\end{aligned}
$$

Apparently, the MRC performs approximately 7 times better than the switched combining technique in terms of BER.