TSDT74 Radio Communication

Solutions to the exam 2014-05-28

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PART A

A.1 The impedance of vacuum is defined as

$$Z_0 = \frac{|E|}{|H|} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi \approx 377\Omega(Ohms).$$

Here $|E|$ and $|H|$ are the strengths of the electric and the magnetic field of an EM-wave, respectively.

A.2 According to the Plane Earth model the power loss for large distances between the receiver and the transmitter is given by

$$L_{PE} \approx \frac{r^4}{(h_1h_2)^2}.$$

Thus the power is 4. For the Free Space propagation model we have

$$L_{FS} \approx \frac{(4\pi r)^2}{\lambda^2},$$

which gives the power 2.

A.3 A WSSUS channel stands for Wide Sense Stationary Uncorrelated Scattering channel. Wide Sense Stationarity means that the impulse response stochastic process $h(\tau;t)$ is WSS with respect to t. Uncorrelated Scattering means that the components of the received signal with different delays are uncorrelated.

The region on the frequency axis where the frequency correlation function $\phi_H(\Delta f) = \phi_H(\Delta f; 0)$ is not equal to zero is called the coherence bandwidth.

The region on the time axis where the time correlation function $\phi_H(\Delta t) = \phi_H(0; \Delta t)$ is not equal to zero is called the coherence time.

A.4 The "effective Earth radius" is defined as the radius for which the radiowave propagation close to the Earth's surfice can be assumed to be a straight line. Due to the fact that the refractive index of the Earth's atmosphere varies with the altitude, the radiowaves close to the Earth's surfice bend towards the surface.

A.5 Examples of space diversity techniques are selection combining, maximal-ratio combining, equal gain combining and switched combining. In selection combining only the strongest branch is chosen. In maximal-ratio combining the signals from all the branches are linearly combined so that the SNR is maximized. The former technique has a simple implementation while the performance is rather poor. The latter technique has more complicated implementation but the performance is superior.

A.6 The OFDM symbol rate is $R_s = 1/T_s = 1/(100 * 10^{-6}) = 10000$ symbols/s. Since 64-QAM have 6 bits/symbol and the OFDM symbol consists of 100 carriers, then the coded bit-rate is $R_{cb} = R_s * 6 * 100 = 6$ Mbits/s. The code rate is $12/24 = 0.5$ thus the information data rate is $R_b = 0.5 * R_{cb} = 3$ Mbits/s.

A.7 The efficiency of a selective-repeat ARQ scheme is given by

$$\eta = \frac{k}{n}P_a = \frac{3}{4}0.9 = 0.675 = 67.5\%$$

The expected message delay is

$$D = \frac{1}{P_a}\frac{n}{r_c} + t_r = \frac{t_p}{P_a} + t_r = \frac{45}{0.9} + 10 = 60 \text{ ms}.$$

A.8 The length of the cyclic prefix for the OFDM symbol is

$$T_G * f_s = 4 \times 10^{-4} \cdot 75 \times 10^3 = 30 \text{ symbols}.$$

A.9 We introduce time-guard interval in wireless communications to avoid inter-symbol interference. The length of this interval should be chosen to be greater than the time delay spread of the channel (T_m).

A.10 In the case of block interleaving we have two equivalent memory blocks at the transmitter and the receiver. In the case of convolutional interleaving we have one common memory block which consists of shift registers of different length. The contents of the registers are consecutively shifted over the channel. The convolutional interleaver has shorter delay.

PART B

B.1 The parameters of the communication link are:

- frequency: $f = 9.8 \cdot 10^8$ Hz;
- transmit power: $[P]_{dB} = 20$ dBW;
- the distance: R m;
- transmit antenna gain: $[G_T]_{dB} = 2$ dB;
- transmited EIRP: $[P_T]_{dB} = [P_T G_T]_{dB} = 22$ dBW;
- receive antenna gain: $[G_R]_{dB} = 2$ dB;
- path loss exponent: 4;
- the median path loss: $[L]_{dB} = 40 \log_{10}\left(\frac{R}{R_0}\right)$;
- receiver noise factor: $[F_R]_{dB} = 6$ dB;
- required SNDR: $[C/N_0]_{dB} \geq 60$ dB-Hz;
- Boltzmann's constant $k = 1.38 \cdot 10^{-23}$ J/K $\Rightarrow [k]_{dB} \approx -228.6$ dB;
- the transmitted power at distance $R_0 = 1$ m:

$$\begin{aligned}
[P_0]_{dB} &= [P_T]_{dB} - [L_p]_{dB} \\
&= [P_T]_{dB} - 20 \log_{10}\left(\tfrac{4\pi R_0}{\lambda}\right) \\
&= [P_T]_{dB} - 20 \log_{10}\left(\tfrac{4\pi R_0 f}{c}\right) \\
&\approx 22 - 32.266 = -10.266\ dBW.
\end{aligned}$$

- receiver's noise temperature:

$$[T_R]_{dB} = [F_R T_0]_{dB} = [F_R]_{dB} + [290]_{dB} \approx 30.624 \text{ dB};$$

The link-budget equation is taking the form:

$$\begin{aligned}
[C/N_0]_{dB} &= [P_0]_{dB} + [G_R]_{dB} - [T_R]_{dB} - [L]_{dB} - [k]_{dB} \\
&= -10.266 + 2 - 30.624 - 40 \log_{10}(R) + 228.6 \\
&= 189.71 - 40 \log_{10}(R).
\end{aligned}$$

Thus the requirement is fulfilled if

$$\begin{aligned}
189.71 - 40 \log_{10}(R) &= [C/N_0]_{dB} \geq 66 \Leftrightarrow \\
123.71 &\geq 40 \log_{10}(R) \Leftrightarrow \\
R &\leq 10^{3.0928} \Leftrightarrow \\
R &\leq 1238.2 \text{ m}.
\end{aligned}$$

B.2

a) The mean-SNR of the channel can be calculated as follows:

$$\begin{aligned}
\int_{-\infty}^{\infty} \gamma dP_\Gamma(\gamma) &= \int_0^\infty \gamma d\left(1 - e^{-\gamma/\gamma_0} - \tfrac{\gamma}{\gamma_0} e^{-\gamma/\gamma_0}\right) \\
&= -\int_0^\infty \gamma d\left(e^{-\gamma/\gamma_0} + \tfrac{\gamma}{\gamma_0} e^{-\gamma/\gamma_0}\right) \\
&= -\gamma\left(e^{-\gamma/\gamma_0} + \tfrac{\gamma}{\gamma_0} e^{-\gamma/\gamma_0}\right)\Big|_0^\infty \\
&\quad + \int_0^\infty \left(e^{-\gamma/\gamma_0} + \tfrac{\gamma}{\gamma_0} e^{-\gamma/\gamma_0}\right) d\gamma \\
&= -\gamma_0 \int_0^\infty \left(1 + \tfrac{\gamma}{\gamma_0}\right) de^{-\gamma/\gamma_0} \\
&= -\gamma_0 \left(1 + \tfrac{\gamma}{\gamma_0}\right) e^{-\gamma/\gamma_0}\Big|_0^\infty \\
&\quad + \gamma_0 \int_0^\infty e^{-\gamma/\gamma_0} d\left(1 + \tfrac{\gamma}{\gamma_0}\right) \\
&= \gamma_0 - \left(\gamma_0 e^{-\gamma/\gamma_0}\right)\Big|_0^\infty \\
&= 2\gamma_0.
\end{aligned}$$

b) The BER for BPSK modulated signal in AWGN channel with SNR of γ is

$$P_b(\gamma) = \mathrm{Q}\left(\sqrt{2\gamma}\right).$$

The BER over a fading channel with SNR following the distribution function $P(\gamma)$ is

$$P_b = E[P_b(\gamma)] = \int_{-\infty}^{\infty} P_b(\gamma) dP_\Gamma(\gamma).$$

In this special case we have

$$\begin{aligned}
P_b &= E[P_b(\gamma)] = \int_{-\infty}^{\infty} \mathrm{Q}\left(\sqrt{2\gamma}\right) dP_\Gamma(\gamma) \\
&= \tfrac{1}{\gamma_0^2} \int_0^\infty \mathrm{Q}\left(\sqrt{2\gamma}\right) \gamma e^{-\gamma/\gamma_0} d\gamma \\
&= \tfrac{1}{\gamma_0^2} \int_0^\infty \left(\int_{\sqrt{2\gamma}}^\infty \tfrac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx\right) \gamma e^{-\gamma/\gamma_0} d\gamma \\
&= \tfrac{1}{\sqrt{2\pi}\gamma_0^2} \int_0^\infty \left(\int_0^{\frac{x^2}{2}} \gamma e^{-\gamma/\gamma_0} d\gamma\right) e^{-\frac{x^2}{2}} dx \\
&= -\tfrac{1}{\sqrt{2\pi}\gamma_0^2} \int_0^\infty \left(\gamma_0(\gamma + \gamma_0) e^{-\gamma/\gamma_0}\Big|_0^{\frac{x^2}{2}}\right) e^{-\frac{x^2}{2}} dx \\
&= \tfrac{1}{\sqrt{2\pi}\gamma_0^2} \int_0^\infty \left(\gamma_0^2 - \gamma_0(\tfrac{x^2}{2} + \gamma_0) e^{-\frac{x^2}{2\gamma_0}}\right) e^{-\frac{x^2}{2}} dx \\
&= \tfrac{1}{2\sqrt{2\pi}\gamma_0^2} \int_{-\infty}^\infty \left(\gamma_0^2 - \gamma_0(\tfrac{x^2}{2} + \gamma_0) e^{-\frac{x^2}{2\gamma_0}}\right) e^{-\frac{x^2}{2}} dx \\
&= \tfrac{1}{2} \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-\frac{x^2}{2}} dx \\
&\quad - \tfrac{1}{2} \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-\frac{\gamma_0+1}{\gamma_0} \frac{x^2}{2}} dx \\
&\quad - \tfrac{1}{4\gamma_0} \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty x^2 e^{-\frac{\gamma_0+1}{\gamma_0} \frac{x^2}{2}} dx \\
&= \tfrac{1}{2} \\
&\quad - \tfrac{1}{2} \sqrt{\tfrac{\gamma_0}{\gamma_0+1}} \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-\frac{x^2}{2}} dx \\
&\quad - \tfrac{1}{4\gamma_0} \tfrac{\gamma_0}{\gamma_0+1} \sqrt{\tfrac{\gamma_0}{\gamma_0+1}} \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty x^2 e^{-\frac{x^2}{2}} dx \\
&= \tfrac{1}{2} - \tfrac{1}{2} \sqrt{\tfrac{\gamma_0}{\gamma_0+1}} - \tfrac{1}{2} \tfrac{1}{2(\gamma_0+1)} \sqrt{\tfrac{\gamma_0}{\gamma_0+1}} \\
&= \frac{1}{2}\left(1 - \frac{2\gamma_0 + 3}{2\gamma_0 + 2} \sqrt{\frac{\gamma_0}{\gamma_0 + 1}}\right)
\end{aligned}$$

c) For the two values of γ_0 we have:

- $\gamma_0 = 26$ dB ≈ 400 and

$$P_b = \frac{1}{2}\left(1 - \frac{2\gamma_0+3}{2\gamma_0+2}\sqrt{\frac{\gamma_0}{\gamma_0+1}}\right) \approx 1.167 \times 10^{-6} > 10^{-6}$$

- $\gamma_0 = 27$ dB ≈ 500 and

$$P_b = \frac{1}{2}\left(1 - \frac{2\gamma_0+3}{2\gamma_0+2}\sqrt{\frac{\gamma_0}{\gamma_0+1}}\right) \approx 7.4751 \times 10^{-7} < 10^{-6}$$

This shows that $\gamma_0 = 27$ dB satisfies the requirement while the value $\gamma_0 = 26$ dB does not satisfy the requirement.

B.3 According to equation (4.142), for non-coherently detected BFSK modulation over a flat Rayleigh fading channel, we can calculate the average BER as:

$$P_b = \frac{1}{2+\gamma_0} = \frac{1}{2+10^{1.6}} \approx \frac{1}{41.8} \approx 0.024$$

The propbability for being in the "bad" state is $q_1/(q_1 + q_2)$, where according to eq. (6.16)

$$\begin{aligned} P_b &= \tfrac{q_1}{q_1+q_2} p_b + \tfrac{q_2}{q_1+q_2} p_g \\ &= \tfrac{q_1}{q_1+q_2} 0.1 + \tfrac{q_2}{q_1+q_2} 0.01 \\ &= \tfrac{10q_1+q_2}{100(q_1+q_2)} = 0.024 \end{aligned}$$

This is equivalent to $q_1 = \frac{7}{38} q_2 \approx 0.184 q_2$.

a) We have that $q_2 = 5\%$ which gives $q_1 \approx 0.92\%$. The probability of acceptance in this case is:

$$\begin{aligned} P_a \approx P_c &= \tfrac{q_1}{q_1+q_2}(1-p_b)^{80} + \tfrac{q_2}{q_1+q_2}(1-p_g)^{80} \\ &= 0.1554 * 0.9^{80} + 0.8446 * 0.99^{80} \approx 0.378 \end{aligned}$$

The efficiency of the selective-repeat scheme is

$$\eta = \frac{k}{n} P_a \approx \frac{3}{4} 0.2835 \approx 28.4\%$$

b) In this case we have uncorrelated errors (like for the AWGN channel) and the probability of acceptance is calculated as

$$P_a \approx P_c = (1 - P_b)^{80} \approx 0.14321.$$

The efficiency in this case is

$$\eta = \frac{k}{n} P_a \approx \frac{60}{80} 0.14321 \approx 10.74\%.$$

B.4 According to equations (5.52) and (5.53), the probability density function of the received SNR after switched combining with threshold γ_x is given by

$$p_\Gamma(\gamma) = \begin{cases} (1+q_x)\dfrac{1}{\gamma_0} e^{-\gamma/\gamma_0}, & \text{if} \quad \gamma \geq \gamma_x, \\ q_x \dfrac{1}{\gamma_0} e^{-\gamma/\gamma_0}, & \text{if} \quad 0 \leq \gamma < \gamma_x, \end{cases}$$

where $q_x = 1 - e^{-\gamma_x/\gamma_0}$. In our case we have $\gamma_0 = \gamma_x = 10$ dB $= 10$ and thus $q_x = 1 - e^{-\gamma_x/\gamma_0} = 1 - e^{-1}$. The BER for the binary DPSK modulation over AWGN channel with SNR γ is given by

$$P_b(\gamma) = \frac{1}{2} e^{-\gamma}.$$

a) The average BER after the switched combining is derived as

$$\begin{aligned} P_b &= E[P_b(\gamma)] = \textstyle\int_0^\infty \frac{1}{2} e^{-\gamma} p_\Gamma(\gamma) d\gamma \\ &= \textstyle\int_0^{\gamma_x} \frac{1}{2} e^{-\gamma} q_x \frac{1}{\gamma_0} e^{-\gamma/\gamma_0} d\gamma \\ &\quad + \textstyle\int_{\gamma_x}^\infty \frac{1}{2} e^{-\gamma} (1+q_x) \frac{1}{\gamma_0} e^{-\gamma/\gamma_0} d\gamma \\ &= \tfrac{q_x}{2\gamma_0} \textstyle\int_0^\infty \gamma e^{-\gamma(\gamma_0+1)/(\gamma_0)} d\gamma \\ &\quad + \tfrac{1}{2\gamma_0} \textstyle\int_{\gamma_x}^\infty \gamma e^{-\gamma(\gamma_0+1)/(\gamma_0)} d\gamma \\ &= \tfrac{q_x}{2\gamma_0} \tfrac{-\gamma_0}{\gamma_0+1} e^{-\gamma(\gamma_0+1)/(\gamma_0)} \Big|_0^\infty \\ &\quad + \tfrac{1}{2\gamma_0} \tfrac{-\gamma_0}{\gamma_0+1} e^{-\gamma(\gamma_0+1)/(\gamma_0)} \Big|_{\gamma_x}^\infty \\ &= \tfrac{q_x}{2(\gamma_0+1)} + \tfrac{e^{-\gamma_x(\gamma_0+1)/(\gamma_0)}}{2(\gamma_0+1)} \\ &= \tfrac{1-e^{-1}+e^{-11}}{22} \approx 0.028732 \approx 2.9\%. \end{aligned}$$

b) According to equation (5.23) the probability density function for the SNR for MRC with two independent Rayleigh fading branches is

$$p_\Gamma(\gamma) = \frac{\gamma}{\gamma_0^2} e^{-\gamma/\gamma_0} \text{if } \gamma \geq 0$$

and 0 otherwise. The average BER when MRC can now be calculated as

$$\begin{aligned} P_b &= E[P_b(\gamma)] = \textstyle\int_0^\infty \frac{1}{2} e^{-\gamma} p_\Gamma(\gamma) d\gamma \\ &= \textstyle\int_0^\infty \frac{1}{2} e^{-\gamma} \frac{\gamma}{\gamma_0^2} e^{-\gamma/\gamma_0} d\gamma \\ &= \tfrac{1}{2\gamma_0^2} \textstyle\int_0^\infty \gamma e^{-\gamma(\gamma_0+1)/(\gamma_0)} d\gamma \\ &= -\tfrac{1}{2\gamma_0(\gamma_0+1)} \gamma e^{-\gamma(\gamma_0+1)/(\gamma_0)} \Big|_0^\infty \\ &\quad \tfrac{1}{2\gamma_0(\gamma_0+1)} \textstyle\int_0^\infty e^{-\gamma(\gamma_0+1)/(\gamma_0)} d\gamma \\ &= -\tfrac{1}{2(\gamma_0+1)^2} e^{-\gamma(\gamma_0+1)/(\gamma_0)} \Big|_0^\infty \\ &= \tfrac{1}{2(\gamma_0+1)^2} = \tfrac{1}{242} \approx 0.0041322 \approx 0.4\%. \end{aligned}$$

Apparently, the MRC performs approximately 7 times better than the switched combining technique in terms of BER.