TSDT74 Radio Communication

Solutions to the exam 2012-06-02

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PART A

A.1 A WSSUS channel stands for Wide Sense Stationary Uncorrelated Scattering channel. Wide Sense Stationarity means that the impulse response stochastic process $h(\tau; t)$ is WSS with respect to t. Uncorrelated Scattering means that the components of the received signal with different delays are uncorrelated.

The length of the region on the frequency axis where the frequency correlation function $\phi_H(\Delta f) = \phi_H(\Delta f; 0)$ is not equal to zero is called the coherence bandwidth.

The length of the region on the time axis where the time correlation function $\phi_H(\Delta t) = \phi_H(0; \Delta t)$ is not equal to zero is called the coherence time.

A.2 Diffraction is the propagation of the EM-waves behind obstacles or around the curved surface of the Earth.

A.3 The time-guard interval for the OFDM symbol have to be greater than the delay (multipath) spread of the WSSUS channel.

A.4 The noise figure F is defined as the ratio $F = T_e/T_0$, where T_e is the system's noise temperature and $T_o = 290K$ is a reference temperature.

A.5 *Half-power bandwidth:* The interval between the two frequencies at which the power spectrum is 3 dB below its peak value.

Equivalent noise bandwidth: Measures the basis of a rectangle whose height is maxf Fs(f) and whose area is one-half of the power of the modulated signal.

Null-to-null bandwidth: The width of the main spectral lobe.

A.6 The bandwidth efficiency of the QPSK modulation is 1 bits/s/Hz. QPSK can not be used with non-coherent detection since the information is conveyed in the phase.

A.7 The mean-SNR of the the diversity system can be calculated as

$$\Gamma = \gamma_1 + \gamma_2.$$

The requirement is satisfied if

$$10^{10/10} + \gamma_2 \geq 10^{20/10},$$

which gives $\gamma_2 \geq 90$ and the least logarithmic value of ≈ 19.54 dB.

A.8 Two techniques are for example the Direct Sequence Spread Spectrum (DS-SS) modulation and the Frequency Hopping Spread Spectrum (FH-SS) modulation. During transmission for the FH-SS, the carrier frequency is changed often and take values from a predefined set of frequencies. This is done by assigning a specific for each user, random-like sequence. Usually this is achieved with the help of PN-sequence generators. Among the advantages of the FH-scheme is the insensitivity to narrow-band interference and wider frequency bandwidth than the DS-SS modulation.

A.9 The efficiency of the "go-back-N" ARQ method is

$$\eta = \frac{k}{n} \times \frac{P_a}{P_a + N(1 - P_a)}.$$

A.10 Interleaving is used to convert a discrete channel with memory into a memoryless channel. The longer the interleaveris the better the conversion.

PART B

B.1 The parameters of the communication link are:

- frequency: $f = 1.5 \cdot 10^9$ Hz;
- wavelength: $\lambda = c/f = 0.2$ m;
- transmit power: $P_T \leq 5$ W ≈ 7 dBW;
- The distance: $R = 1.5 \cdot 10^5$ m;
- transmit antenna gain : $[G_T]_{dB} = 15$ dB;
- receive antenna gain (dipole): $[G_R]_{dB} = 2.1$ dB;
- receiver noise figure: $[F_R]_{dB} = 25$ dB≈ 316.23;
- receiver's feeder loss: $[L_{RF}]_{dB} = 0$ dB;
- transmitter's feeder loss: $[L_{TF}]_{dB} = 3$ dB;
- required SNR: $[S/N]_{dB} \geq 30$ dB;
- Boltzmann's constant $k = 1.38 \cdot 10^{-23}$ J/K $\Rightarrow [k]_{dB} \approx -228.6$ dB;

From these parameters we need to calculate:

- the free space loss: $[L_{FS}]_{dB} = \left[\left(\frac{4\pi R}{\lambda}\right)^2\right]_{dB} \approx$ 139.49 dB;
- receiver's noise temperature: $[T_R]_{dB} = [F_R T_0]_{dB} = [290 F_R]_{dB} = 49.624$ dB;

The link-budget equation is taking the form:

$$\begin{aligned} [S/N]_{dB} &= [P_T]_{dB} + [G_T]_{dB} + [G_R]_{dB} - [T_R]_{dB} \\ &\quad -[L_{FS}]_{dB} - [L_{TF}]_{dB} - [L_{RF}]_{dB} - [B]_{dB} - [k]_{dB} \\ &\leq 7 + 15 + 2.1 - 49.624 \\ &\quad -139.49 - 3 - 0 - [B]_{dB} + 228.6 \\ &= 60.586 - [B]_{dB}. \end{aligned}$$

Thus the requirement is fulfilled if

$$\begin{aligned} 60.586 - [B]_{dB} &\geq [C/N]_{dB} \geq 30 \Leftrightarrow \\ 30.586 &\geq [B]_{dB} \Leftrightarrow \\ B &\leq 1144.5 \text{ Hz}. \end{aligned}$$

B.2 a) For the received signals we have that

$$\begin{aligned} r_i(t) &= (h * s_i)(t) = \Re\left\{h_0 m(t-0.1)e^{j2\pi f_i(t-0.1)}\right. \\ &\qquad \left. + h_1 m(t-0.6)e^{j2\pi f_i(t-0.6)}\right\}. \end{aligned}$$

Thus the equivalent low-pass of the received signals are given as

$$\tilde{r_i}(t) = h_0 m(t-0.1)e^{j2\pi f_i(t-0.1)} + h_1 m(t-0.6)e^{j2\pi f_i(t-0.6)},$$

where the time t is given in μs.

b) The intesity profile is calculated as follows

$$\begin{aligned} \varphi_h(\tau) &= \varphi_h(\tau; 0) \\ &= E\{h(\tau;t)h^*(\tau;t)\} \\ &= E\{(h_0\delta(\tau-0.1) + h_1\delta(\tau-0.6))\times \\ &\quad \times(h_0^*\delta(\tau-0.1) + h_1^*\delta(\tau-0.6))\} \\ &= E\{|h_0|^2\delta(\tau-0.1) + |h_1|^2\delta(\tau-0.6)\} \\ &= E\{|h_0|^2\}\delta(\tau-0.1) + E\{|h_1|^2\}\delta(\tau-0.6). \end{aligned}$$

c) The delay spread is $T_M = 0.6 - 0.1 = 0.5\mu$s. The coherence bandwidth is $B_M \approx 1/T_M = 2MHz$. Thus the minimum frequencey separation is $2MHz$.

B.3

a) We have that $\gamma_0 = 6$ dB ≈ 4. According to eq. (5.23) the probability density function of the received SNR after maximum-ratio combining with $M = 2$ branches is given by

$$p_\Gamma(\gamma) = \begin{cases} \frac{\gamma}{\gamma_0^2} e^{-\gamma/\gamma_0}, & \text{if} \quad \gamma \geq 0, \\ 0, & \text{if} \quad \gamma < 0, \end{cases}$$

The BER for non-coherent BFSK modulation over AWGN channel with SNR γ is given by

$$P_b(\gamma) = \frac{1}{2}e^{-\gamma/2}.$$

The BER after the maximum ratio combining is derived as

$$\begin{aligned} P_b &= E[P_b(\gamma)] = \int_{-\infty}^{\infty} \tfrac{1}{2}e^{-\gamma/2} p_\Gamma(\gamma)d\gamma \\ &= \int_0^\infty \tfrac{1}{2}e^{-\gamma/2}\tfrac{\gamma}{\gamma_0^2}e^{-\gamma/\gamma_0}d\gamma \\ &= \tfrac{1}{2\gamma_0^2}\int_0^\infty \gamma e^{-\gamma(\gamma_0+2)/(2\gamma_0)}d\gamma \\ &= \tfrac{-1}{\gamma_0(\gamma_0+2)}\left(\gamma e^{-\gamma(\gamma_0+2)/(2\gamma_0)}\Big|_0^\infty\right. \\ &\quad \left. - \int_0^\infty \gamma e^{-\gamma(\gamma_0+2)/(2\gamma_0)}d\gamma\ \right) \\ &= \tfrac{-2}{(\gamma_0+2)^2}\ e^{-\gamma(\gamma_0+2)/(2\gamma_0)}\Big|_0^\infty \\ &= \tfrac{2}{(\gamma_0+2)^2} \end{aligned}$$

Substituting $\gamma_0 = 6$ dB ≈ 4 we obtain the average BER

$$P_b \approx 0.05556 = 5.56\%.$$

b)

The probability density function of the received SNR after switched combining with threshold γ_x is given by

$$p_\Gamma(\gamma) = \begin{cases} (1+q_x)\frac{1}{\gamma_0}e^{-\gamma/\gamma_0}, & \text{if} \quad \gamma \geq \gamma_x, \\ q_x \frac{1}{\gamma_0}e^{-\gamma/\gamma_0}, & \text{if} \quad 0 \leq \gamma < \gamma_x, \end{cases}$$

where $q_x = 1 - e^{-\gamma_x/\gamma_0}$. The BER for non-coherent BFSK modulation over AWGN channel with SNR γ is given by

$$P_b(\gamma) = \frac{1}{2}e^{-\gamma/2}.$$

The BER after the switched combining is derived as

$$\begin{aligned} P_b &= E[P_b(\gamma)] = \int_0^\infty \tfrac{1}{2}e^{-\gamma/2}p_\Gamma(\gamma)d\gamma \\ &= \int_0^{\gamma_x} \tfrac{1}{2}e^{-\gamma/2}q_x\tfrac{1}{\gamma_0}e^{-\gamma/\gamma_0}d\gamma \\ &\quad + \int_{\gamma_x}^\infty \tfrac{1}{2}e^{-\gamma/2}(1+q_x)\tfrac{1}{\gamma_0}e^{-\gamma/\gamma_0}d\gamma \\ &= \tfrac{q_x}{2\gamma_0}\int_0^\infty \gamma e^{-\gamma(\gamma_0+2)/(2\gamma_0)}d\gamma \\ &\quad + \tfrac{1}{2\gamma_0}\int_{\gamma_x}^\infty \gamma e^{-\gamma(\gamma_0+2)/(2\gamma_0)}d\gamma \\ &= \tfrac{q_x}{2\gamma_0}\tfrac{-2\gamma_0}{\gamma_0+2}\; e^{-\gamma(\gamma_0+2)/(2\gamma_0)}\Big|_0^\infty \\ &\quad + \tfrac{1}{2\gamma_0}\tfrac{-2\gamma_0}{\gamma_0+2}\; e^{-\gamma(\gamma_0+2)/(2\gamma_0)}\Big|_{\gamma_x}^\infty \\ &= \tfrac{q_x}{\gamma_0+2} + \tfrac{e^{-\gamma_x(\gamma_0+2)/(2\gamma_0)}}{\gamma_0+2} \\ &= \tfrac{1-e^{-\gamma_x/\gamma_0}+e^{-\gamma_x(\gamma_0+2)/(2\gamma_0)}}{\gamma_0+2}. \end{aligned}$$

Substituting the values $\gamma_0 = 6$ dB ≈ 4 and $\gamma_x = 10$ dB $= 10$ we obtain the average BER

$$P_b \approx 0.15308 = 15.308\%.$$

c) To find the value of the threshold γ_x that minimizes the average BER P_b we have to find a root of the derivative of P_b with respect to γ_x. It is calculated as

$$\begin{aligned} \frac{dP_b}{d\gamma_x} &= \frac{d\frac{1-e^{-\gamma_x/\gamma_0}+e^{-\gamma_x(\gamma_0+2)/(2\gamma_0)}}{\gamma_0+2}}{d\gamma_x} \\ &= \frac{\frac{1}{\gamma_0}e^{-\gamma_x/\gamma_0} - \frac{\gamma_0+2}{2\gamma_0}e^{-\gamma_x(\gamma_0+2)/(2\gamma_0)}}{\gamma_0+2} \\ &= \frac{2e^{-\gamma_x/\gamma_0} - (\gamma_0+2)e^{-\gamma_x(\gamma_0+2)/(2\gamma_0)}}{2\gamma_0(\gamma_0+2)}. \end{aligned}$$

Solving $\frac{dP_b}{d\gamma_x} = 0$ is equivalent to

$$2e^{-\gamma_x/\gamma_0} - (\gamma_0+2)e^{-\gamma_x(\gamma_0+2)/(2\gamma_0)} = 0,$$

which gives

$$\gamma_x = 2\ln((\gamma_0+2)/2) \approx 2.1972 \approx 3.42 \text{ dB}.$$

for this value of γ_x we get

$$P_b \approx 0.10252 = 10.252\%.$$

B.4

We have that $\gamma_0 = 23$ dB ≈ 200. The BER for DBPSK in Rayleigh fading is

$$P_b = \frac{1}{2(1+\gamma_0)} \approx \frac{1}{402} \approx 0.0024876.$$

The number of information bits in a message is $k = 150 - 24 = 126$ and the message length is $n = 150$ bits.

a) In the case of fast fading, the channel can be considered as memoryless with transition probability P_b and thus the probability of acceptance of a message is

$$P_a \approx P_c = (1-P_b)^{150} \approx 0.68825.$$

The efficiency in this case is

$$\eta = \frac{k}{n}P_a \approx 0.57813.$$

b) As in a) the probability of acceptance of a message is

$$P_a \approx P_c = (1-P_b)^{150} \approx 0.68825.$$

The efficiency in this case is

$$\eta = \frac{k}{n}\frac{P_a}{1+t_ir_c/n} \approx 0.542.$$

c) According to equation (6.16) we have

$$P_b = \frac{q_1}{q_1+q_2}p_b + \frac{q_2}{q_1+q_2}p_g.$$

If we set $x = \frac{q_1}{q_1+q_2}$ and use the facts that $\frac{q_2}{q_1+q_2} = 1 - x$, $p_b = 0.01$, $p_g = 0.001$ and $P_b \approx 0.0025$ we get the equiation

$$0.0025 = 0.01x + 0.001(1-x),$$

which gives $x \approx 0.166667$. This is the probability for being in the bad state. The probability of acceptance in the bad state is

$$P_{ab} \approx P_c = (1-p_b)^{150} \approx 0.22145,$$

while the probability of acceptance in the good state is

$$P_{ag} \approx P_c = (1 - p_c)^{150} \approx 0.86064.$$

Now the probability for acceptance is

$$P_a = x * P_{ab} + (1 - x) * P_{ag} \approx 0.75411$$

Finally, the efficiency in this case is

$$\eta = \frac{k}{n} P_a \approx 0.63345.$$