LINKÖPINGS UNIVERSITET

Exam in TSDT16 Felrättande koder

Date: 2014-10-31 **Time:** 14.00-18.00

Teacher: Danyo Danev, tel: 013-281335

Place: G32

Allowed aids: No aids allowed except pocket calculators.

Language: Solutions in both english and swedish are accepted.

Grading: Each correctly answered question yields 3 points. For grade 3 you need 6 points, for grade 4 you need 9 points, for grade 5 you need 12 points. The ECTS levels are as follows: E − > 6, D − > 7, C − > 9, B − > 10, A − > 12. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

Solutions: Will be published after the exam at http://www.commsys.isy.liu.se/en/student/kurser/tentor?TSDT16

Grading list: A preliminary grading list will be send to all registered for the exam no later than 2014-11-10. Others can get information about the results from the course leader or at the exam return.

Exam return: 2014-11-10, kl. 12.00-12.30 in Täljaren, house B, entrance 29, 1st floor.

Complaints: No later than 2014-11-10.

Good luck!

1 The double error-correcting RS code C of length 15 over $\mathbf{GF}(16)$ is is defined by the generator polynomial

$$\mathbf{g}(X) = (X+\alpha)(X+\alpha^2)(X+\alpha^3)(X+\alpha^4),$$

where $\alpha \in \mathbf{GF}(16)$ is a primitive element and a root of the polynomial $X^4 + X^3 + 1$. Create a table utilizing easy computations in $\mathbf{GF}(16)$ based on α and find the codeword in C which is closest, in terms of Hamming distance, to the received vector polynomial $\mathbf{r}(X) = X^{10} + \alpha^4 X^8 + \alpha^{10} X^6$.

2 Let C_1 and C_2 be binary linear cyclic codes of length 15. These codes have generator polynomials

$$\mathbf{g_1}(X) = X^5 + X^4 + X^2 + 1 \text{ and } \mathbf{g_2}(X) = X^5 + X^3 + X + 1.$$

Show that the intersection $C = C_1 \cap C_2$ is a binary linear cyclic code. Find the generator polynomial $\mathbf{g}(X)$ of C as well as its dimension and minimum distance.

3 Consider the binary cyclic code C of length 17 that has generator polynomial

$$\mathbf{g}(X) = X^9 + X^8 + X^6 + X^3 + X + 1.$$

a) Calculate the parity-check polynomial $\mathbf{h}(X)$ of this code. (1 p)

b) Design syndrome computation and encoding circuits for C. (1 p)

c) Find the generator matrix G and the parity-check matrix H of C in systematic form. (1 p)

4 Provide a generator polynomial of the least possible degree for a double-error-correcting BCH-code of length $n = 26$ over över $\mathbf{GF}(3)$. The code should be cyclic and the generator polynomyal $\mathbf{g}(X) \in \mathbf{GF}(3)[X]$ should be divisible by the primitive polynomial $\mathbf{p}(X) = X^3 + 2X + 1 \in \mathbf{GF}(3)[X]$.

5 We use a discrete memoryless channel (DMC) to transmit a binary message $\mathbf{m}$ of length 5. The DMC has two inputs and four outputs and the soft information can be modeled by the following integer metric

	$\mathbf{0_1}$	$\mathbf{0_2}$	$\mathbf{1_2}$	$\mathbf{1_1}$
0	5	4	1	0
1	0	1	4	5

.

In order to increase the link reliability a convolutional code of rate $R = 1/2$ with the generator matrix

$$\mathbf{G}(D) = \left[1 + D + D^2, D + D^2\right]$$

has been used. At the receiver end the following vector has been provided to the Viterbi decoder

$$\mathbf{r} = (1_1 0_2, 1_1 1_1, 0_2 0_1, 0_2 1_1, 0_1 1_2, 1_1 1_2, 1_2 0_1).$$

Determine the message $\mathbf{m}$ that this decoder will provide as maximum-likelihood message transmitted. Assume that the encoder is put into the initial all-zero state after the encoding of the message.

Some useful equations

$$d_\mu = S_{\mu+1} + \sigma_1^{(\mu)} S_\mu + \sigma_2^{(\mu)} S_{\mu-1} + \cdots + \sigma_{l_\mu}^{(\mu)} S_{\mu+1-l_\mu}$$

$$\sigma^{(\mu+1)}(X) = \sigma^{(\mu)}(X) - d_\mu d_\rho^{-1} X^{\mu-\rho} \sigma^{(\rho)}(X)$$