LINKÖPINGS UNIVERSITET

Exam in TSDT16/TEN1 Felrättande koder

Date: 2011-10-22 **Time:** 14.00-18.00

Teacher: Danyo Danev, tel: 0733-213183

Place: U14

Allowed aids: No aids allowed except pocket calculators.

Language: Solutions in both english and swedish are accepted.

Grading: Each correctly answered question yields 3 points. For grade 3 you need 6 points, for grade 4 you need 9 points, for grade 5 you need 12 points. The ECTS levels are as follows: E − > 6, D − > 7, C − > 9, B − > 10, A − > 12. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

Solutions: Will be published after the exam at http://www.commsys.isy.liu.se/en/student/kurser/tentor?TSDT16

Grading list: A preliminary grading list will be send to all registered for the exam no later than 2011-11-04. Others can get information about the results from the course leader or at the exam return.

Exam return: 2011-11-07, kl. 12.15-13.00 in Hammingrummet, house B, entrance 29, 2nd floor.

Complaints: No later than 2011-11-07.

Good luck!

1 Let C be the primitive double error-correcting BCH code of length 15 over $\mathbf{GF}(4) = \{0, 1, \beta, \beta^2\}$. The code C is used for information transmission and the received vector is

$$\mathbf{r} = (\beta^2, 1, 0, 0, 1, 0, \beta^2, 0, 0, 0, 0, \beta, 1, 0, 0).$$

Find the codeword in C which is closest, in terms of Hamming distance, to this received vector. Use the fact that the polynomial $1 + X + X^4$ is primitive over $\mathbf{GF}(2)$.

2 Calculate the minimum distance of the Gallager LDPC code with a parity-check matrix given below. Motivate your answers!

$$H = \begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

3 Design encoding and syndrome computation circuits for the binary primitive double error-correcting BCH-code of length 15. Compute the syndrome of the received polynomial $\mathbf{r}(X) = 1 + X^3 + X^5 + X^9 + X^{12}$ and show the contents of the syndrome register after each shift of $\mathbf{r}$ into the circuit.

4 A binary message $\mathbf{m}$ of length 4 has been transmitted over a discrete memoryless channel for which the following metric table can be obtained.

	$\mathbf{0_1}$	$\mathbf{0_2}$	$\mathbf{1_2}$	$\mathbf{1_1}$
0	5	4	1	0
1	0	1	4	5

In order to decrease the probability of error, a rate one third convolutional code with generator matrix

$$\mathbf{G}(D) = \left[D + D^2, 1 + D, 1 + D + D^2\right]$$

is used. The demodulator provides the vector

$$\mathbf{r} = (1_1 0_1 1_2, 0_1 1_2 1_1, 1_1 0_2 1_2, 1_1 1_2 0_1, 1_2 0_1 0_1, 1_1 1_2 1_1).$$

Decode the transmitted message $\mathbf{m}$ with a maximum-likelihood decoding algorithm. Assume that two "termination" bits that put the encoder in initial "zero" state, have been added to the message prior to encoding.

5 Let C be a binary cyclic code of length n with generator polynomial $\mathbf{g}(X)$ and weight enumerator $A(z)$. Show that if there is a codeword in C of odd weight, then the weight enumerator $A_1(z)$ of the cyclic code C_1 of length n with generator polynomial $\mathbf{g_1}(X) = (1 + X)\mathbf{g}(X)$ can be calculated as follows

$$A_1(z) = \frac{A(z) + A(-z)}{2}.$$