Exam in TSDT16/TEN1 Felrättande koder

Date: 2010-10-23 **Time:** 14.00-18.00

Teacher: Danyo Danev, tel: 0733-213183

Place: TER4

Allowed aids: No aids allowed except pocket calculators.

Language: Solutions in both english and swedish are accepted.

Grading: Each correctly answered question yields 3 points. For grade 3 you need 6 points, for grade 4 you need 9 points, for grade 5 you need 12 points. The ECTS levels are as follows: E − > 6, D − > 7, C − > 9, B − > 10, A − > 12. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unreasonable answers.

Solutions: Will be published after the exam at http://www.commsys.isy.liu.se/en/student/kurser/tentor?TSDT16

Grading list: A preliminary grading list will be send to all registered for the exam no later than 2010-11-05. Others can get information about the results from the course leader or at the exam return.

Exam return: 2010-11-08, kl. 12.30-13.00 in Hammingrummet, house B, entrance 29, 2nd floor.

Complaints: No later than 2010-11-08.

Good luck!

1 Let us consider two linear block codes C_1 and C_2. The code C_i has length n_i, dimension k, minimum distance d_i and is defined by the systematic generator matrix $\mathbf{G_i} = [\mathbf{P_i}|\mathbf{I_k}]$ for $i = 1, 2$. We construct a new linear block code C which has the parity check matrix

$$\mathbf{H} = \left[\begin{array}{c|c} & \mathbf{P_1^T} \\ \mathbf{I_{n_1+n_2-k}} & \mathbf{I_k} \\ & \mathbf{P_2^T} \end{array} \right].$$

What is the dimension and the length of the code C? Show also that C has a minimum distance of at least $d_1 + d_2$!

2 The triple error-correcting RS code C has length 15. The generator polynomial of C is

$$\mathbf{g}(X) = (X + \alpha)(X + \alpha^2)(X + \alpha^3)(X + \alpha^4)(X + \alpha^5)(X + \alpha^6),$$

where $\alpha \in \mathbf{GF}(16)$ is a primitive element satisfying the equation $1 + \alpha + \alpha^4$. Create the table needed for the computations in $\mathbf{GF}(16)$ based on α. If the received vector $\mathbf{r}$ is represented by the polynomial

$$\mathbf{r}(X) = X^6 + \alpha^4 X^3 + \alpha^9 X + \alpha^6,$$

determine the codeword in C which is closest, in terms of Hamming distance, to $\mathbf{r}$.

3 Devise a syndrome computation and encoding circuits for the binary cyclic code C of length 15 which has the parity polynomial

$$\mathbf{h}(X) = X^6 + X^4 + X^3 + X^2 + 1.$$

Compute the syndrome of the received polynomial $\mathbf{r}(X) = 1 + X^6 + X^{10}$ and show the contents of the syndrome register after each shift of $\mathbf{r}$ into the circuit.

4 The convolutional code of rate $R = 2/3$ with a generator matrix

$$\mathbf{G}(D) = \begin{bmatrix} 1+D & D & 1+D \\ D & 1 & 1+D \end{bmatrix}.$$

has been used to decrease the BER when transmitting binary information over a discrete memoryless channel (DMC). The demodulator provides two versions of the transmitted binary symbols. The integer metric table that can be obtained based on the corresponding probabilities is

	$\mathbf{0_1}$	$\mathbf{0_2}$	$\mathbf{1_2}$	$\mathbf{1_1}$
0	10	7	2	0
1	0	2	7	10

After passing the DMC the result from the demodulator is the following vector

$$\mathbf{r} = (1_2 0_1 1_1, 0_1 1_2 0_2, 1_2 1_1 1_1, 0_1 0_1 0_1).$$

Determine the message $\mathbf{m}$ that has been sent if a maximum-likelihood decoding algorithm is used. Assume that the encoder is put into the initial all-zero state after the encoding of the message.

5 Let C be the binary Hamming code of length 15 and dimension 11. The code C is used for error detection of messages transmitted over a memoryless binary symmetric channel with transition probability $p = 0.1$.

a) Find the weight distribution of the code C. (2p)

b) Calculate the probability of undetected error for one packet of length 15. (1p)