Solutions to Exam in Digital Filters 180530

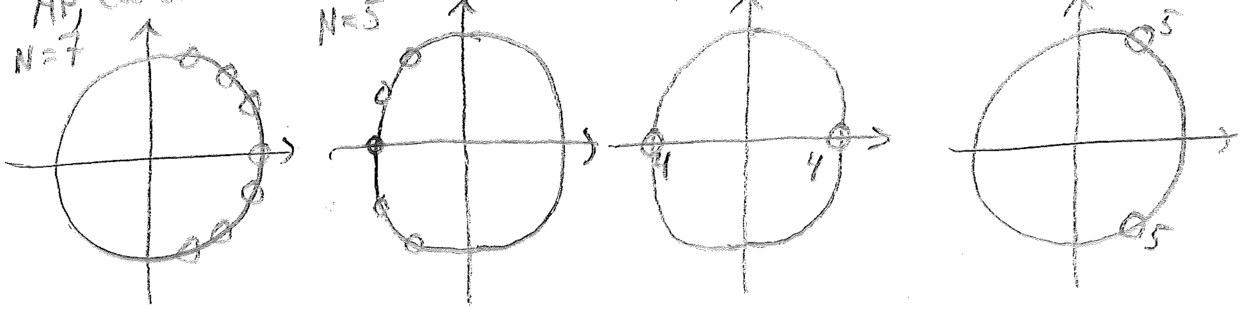
1a) High sensitivity - Larger variations in $H(z)$ when coefficients vary ⇒ longer coeff. wordlength

b) Inputs to noninteger mult., and output are the critical nodes
safe scaling - never overflows
L_p-norm scaling - overflows with certain probability

c) To avoid delay-free loops

d) Symmetric impuls response around $n=3$ ∴ Linear phase

e) HP, Cauer N=7 | LP, Ch-II N=5 | BP, Ch-I N=8 | BS, Butt.w. N=16



2) Edges of analog BP filter (use $\frac{2}{T}=1$)

$\omega_{ac1} = \tan\left(\frac{0{,}375\pi}{2}\right) = 0{,}6682$, $\omega_{ac2} = \tan\left(\frac{0{,}625\pi}{2}\right) = 1{,}4966$

$\omega_{as1} = \tan\left(\frac{0{,}125\pi}{2}\right) = 0{,}1989$, $\omega_{as2} = \tan\left(\frac{0{,}875\pi}{2}\right) = 5{,}0273$

BP → LP: $\omega_{ac1}\omega_{ac2} \overset{\text{OK!}}{=} \omega_{as1}\omega_{as2} = \omega_I^2 = 1$

$\Omega_{ac} = \omega_{ac2} - \omega_{ac1} = 0{,}8284$
$\Omega_{as} = \omega_{as2} - \omega_{as1} = 4{,}8284$
$\Rightarrow \frac{\Omega_{as}}{\Omega_{ac}} = 5{,}83$

Filter order $N=2$

Normalized poles $P'_{LP} = -0{,}7128 \pm j1{,}0040$

zeros Two at $s=\infty$

Denormalize (Mult. by Ω_{ac}) ⇒

Denormalized poles: $P_{a\ell p} = -0{,}5905 \pm j\,0.8318$

zeros: Two at $\mathcal{S} = \infty$

<u>LP → BP</u>: $\mathcal{S} = s + \frac{\omega_T^2}{s} = |\omega_T^2 = 1| = s + \frac{1}{s} = \frac{s^2+1}{s}$

$\Leftrightarrow s^2 - \mathcal{S} \cdot s + 1 = 0 \Leftrightarrow$

$s = \frac{\mathcal{S}}{2} \pm \sqrt{\left(\frac{\mathcal{S}}{2}\right)^2 - 1} \Rightarrow$

poles: $-0.1782 \pm j\,0.6327$

$-0.4124 \pm j\,1.4645$

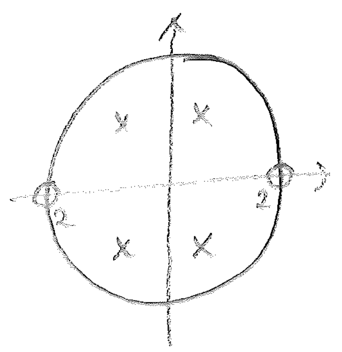
zeros: Two at $s = \infty$, Two at $s = 0$

<u>s → z</u> $z = \frac{1+s}{1-s} \quad \left(\frac{2}{T} = 1\right) \Rightarrow$

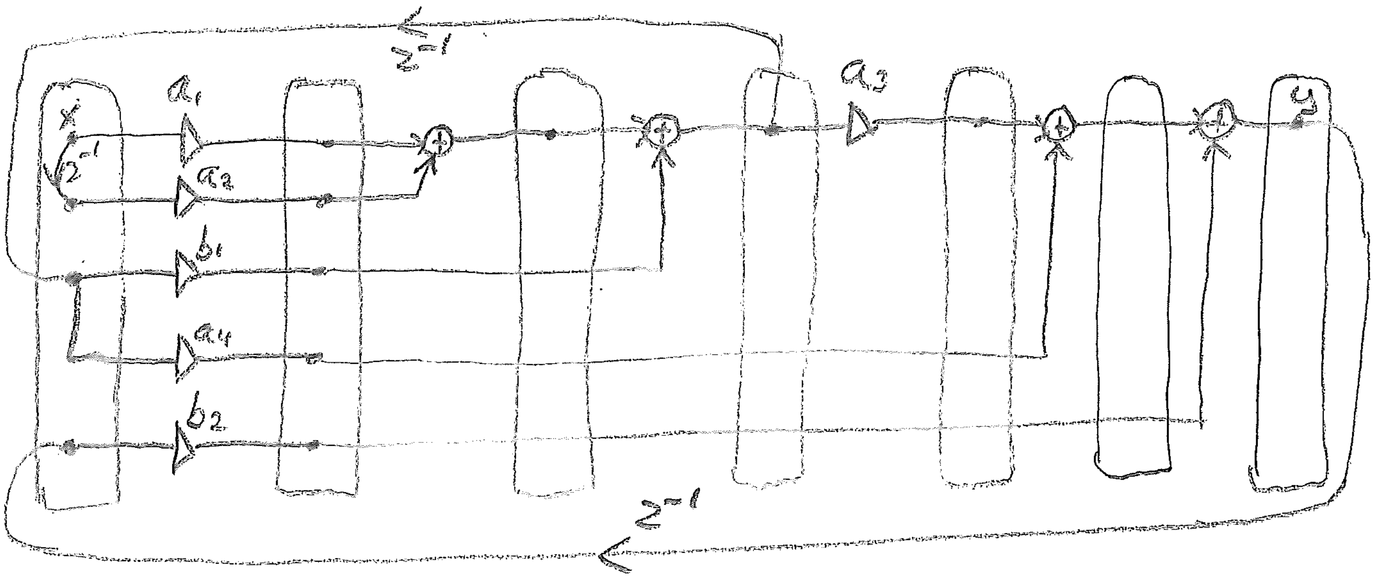
poles: $-0.3176 \pm j\,0.7076$

$0.3176 \pm j\,0.7076$

zeros: Two at $z = 1$, Two at $z = -1$

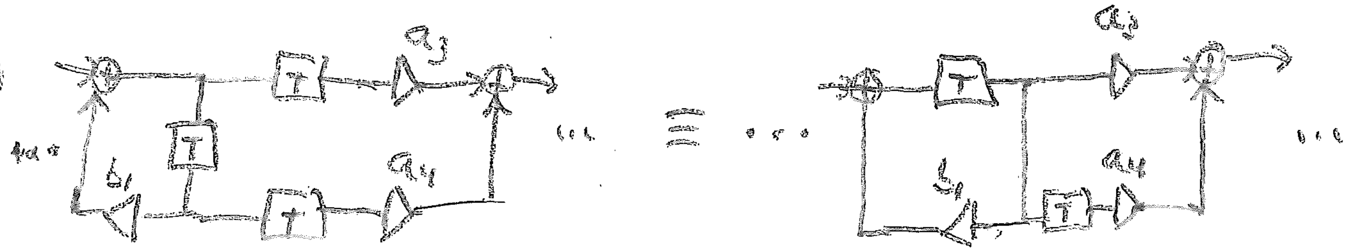


3a)



b) $T_{min} = T_{mult} + T_{add}$ (Two loops, each with one mult, and one add.)

c)



4a) $P_{ye} = P_{ye_1} + P_{ye_2} + P_{ye_3}$, $P_e = \frac{Q^2}{12}$

From b   a   -a

$$P_{ye_1} = P_e \cdot \underbrace{\sum_{n=0}^{\infty} b^{2n}}_{} = P_e \cdot \frac{1}{1-b^2}$$

since second section is allpass whic does not affect the noise gain

$$P_{ye_2} = P_{ye_3} = P_e \cdot \sum_{n=0}^{\infty} a^{2n} = P_e \cdot \frac{1}{1-a^2}$$

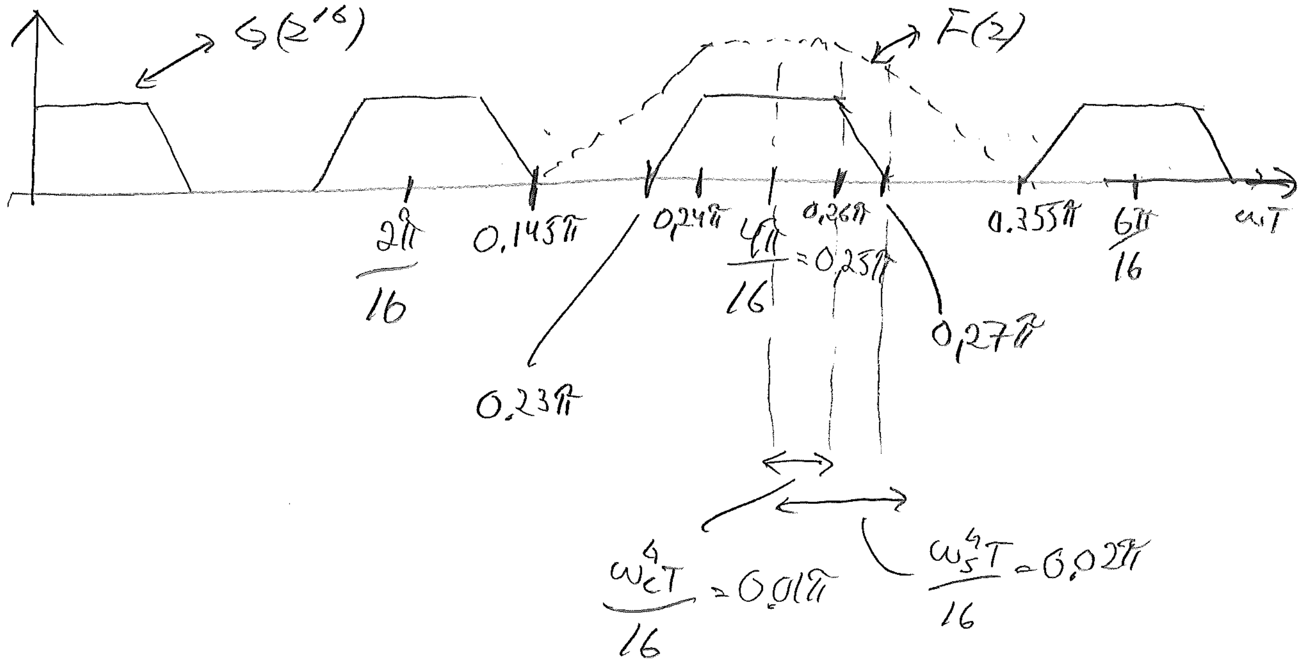
$$\Rightarrow P_{ye} = P_e \cdot \left(\frac{1}{1-b^2} + \frac{2}{1-a^2} \right) = 1,9711 \cdot 10^{-8}$$

b) Due to the allpass section, which does not affect the L_2-norm, it is sufficient to scale the input to -a-mult, because then also the output is scaled as well as the inputs to b-mult. and a-mult

Insert mult. c at the input of the filter

$$c = \frac{1}{\sqrt{\sum_{n=0}^{\infty} b^{2n}}} = \frac{1}{\sqrt{\frac{1}{1-b^2}}} = \sqrt{1-b^2} = 0.7806$$

6a)



From above: $\omega_c^G T = 0.16\pi$, $\omega_s^G = 0.32\pi$

$\omega_{c_1}^F T = 0{,}24\pi$, $\omega_{c2}^F T = 0{,}26\pi$

$\omega_{s1}^F T = 0{,}145\pi$, $\omega_{s2}^F T = 0.355\pi$

(In general $\delta_c^F + \delta_c^G = \delta_c^H$)

Ripples: passband (Ex) 0.005, stopband 0.001 } for both F & G

b) $H(z)$, one filter, transition band $= 0.01\pi$, $\delta_c = 0.01$, $\delta_s = 0.001$

$\Rightarrow N \approx 518 \Leftrightarrow \frac{N}{2}+1 = 260$ mult.

$G(z)$, transit. band $= 0.16\pi$ ripples from a/

$F(z)$, –''– $\approx 0.095\pi$, –''– } $\Rightarrow$

$\Rightarrow N_G \approx 34 \Leftrightarrow 18$ mult

$N_F \approx 58 \Leftrightarrow 30$ mult

In total 48 mult., 80% reduction from 260

7) $$H(e^{j\omega T}) = \frac{e^{j\Phi_0(\omega T)} + e^{j\Phi_1(\omega T)}}{2}$$

$$= e^{j\frac{\Phi_0(\omega T)+\Phi_1(\omega T)}{2}} \cdot \cos\left(\frac{\Phi_0(\omega T) - \Phi_1(\omega T)}{2}\right)$$

$$H_c(e^{j\omega T}) = /+ \to -/ = e^{j\frac{\Phi_0(\omega T)+\Phi_1(\omega T)}{2}} \cdot j\sin\left(\frac{\Phi_0(\omega T) - \Phi_1(\omega T)}{2}\right)$$

a) From above: $|H|^2 + |H_c|^2 = \cos^2(\cdot) + \sin^2(\cdot) = 1$

b) — " — : $\Phi(\omega T) = \frac{\Phi_0(\omega T) + \Phi_1(\omega T)}{2} + \left(\pm\pi + \pi/2\right)$ (if cos, sin change sign, for H_c) Phase discont. ignored here

$$\Rightarrow \tau_g(\omega T) = \frac{\tau_{g0}(\omega T) + \tau_{g1}(\omega T)}{2}$$

c) $-z^2 \leftrightarrow -e^{j2\omega T} = e^{j2\omega T \pm \pi}$

Corresponds to frequency shift by $\pm\pi$ followed by compression by two ($\omega T \to 2\omega T$)

$\therefore$ $H(-z^2) \Leftrightarrow$ Bandpass (if $H(z)$ lowpass)

$H_c(-z^2) \Leftrightarrow$ Bandstop (if $H_c(z)$ highpass)