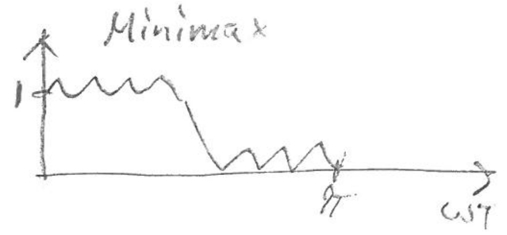
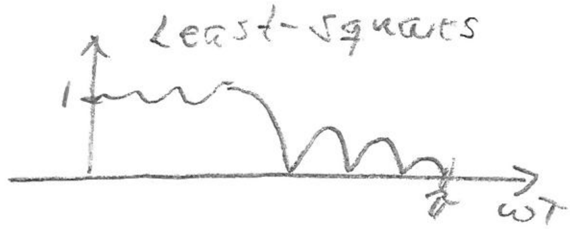
Digital Filters 170603 - Solutions [redacted] TSTE06

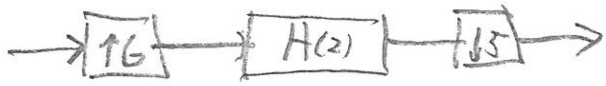
1a) To avoid delay-free loops

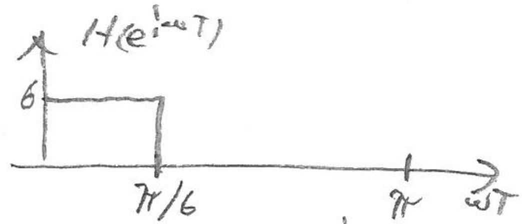
b)





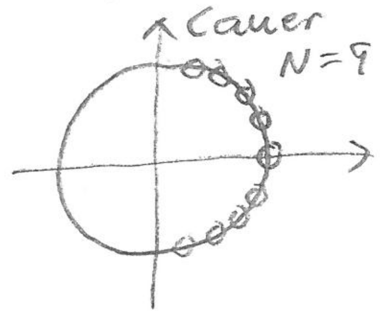
c)

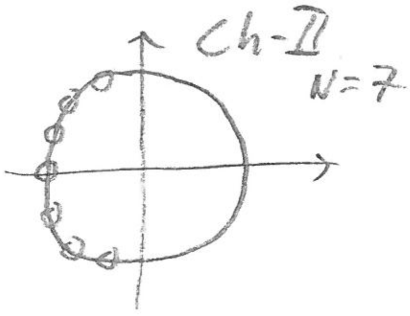


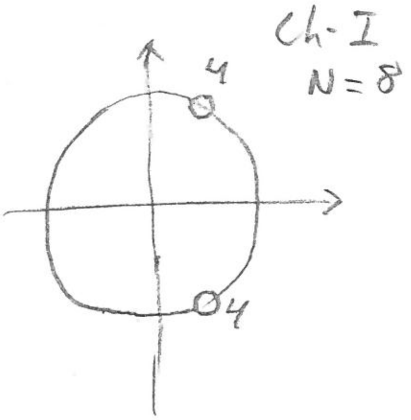


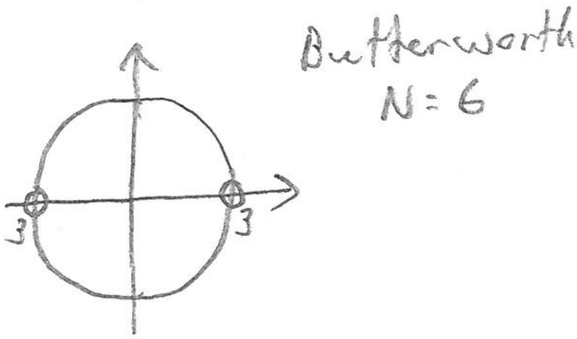
d) Type I ($j \cdot H_{R_1}(\omega T) \cdot j \cdot H_{R_2}(\omega T) = -H_{R_1}(\omega T) H_{R_2}(\omega T)$)

e)









2) Spec. of analog filter (Here $\frac{2}{T}=1$ used)

$\omega_{ac} = \tan\left(\frac{\omega_c T}{2}\right) = 3.0777$, $\omega_{as} = \tan\left(\frac{\omega_s T}{2}\right) = 1$

HP → LP spec.

$\Omega_c = \frac{\omega_I^2}{\omega_{ac}} = \left|\text{use } \omega_I^2 = \omega_{ac}\right| = 1$, $\Omega_s = \frac{\omega_{ac}}{\omega_{as}} = 3.0777$

Filter order $N=4$ (Ch.-I, $\frac{\Omega_s}{\Omega_c} = 3,0777$)

Denormalized poles & zeros = denormalised since $\Omega_c = 1$

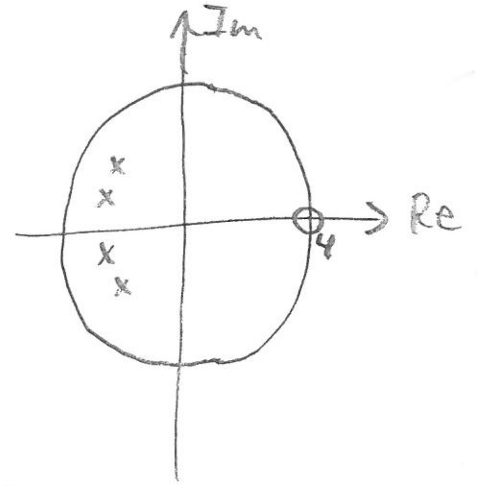
LP: zeros: 4 at ∞
poles: $-0,1754 \pm j1.0163$
$-0,4233 \pm j0,4209$
} S-domain

LP → HP $\quad S = \frac{\omega_I^2}{s} = \frac{\omega_{ac}}{s}$

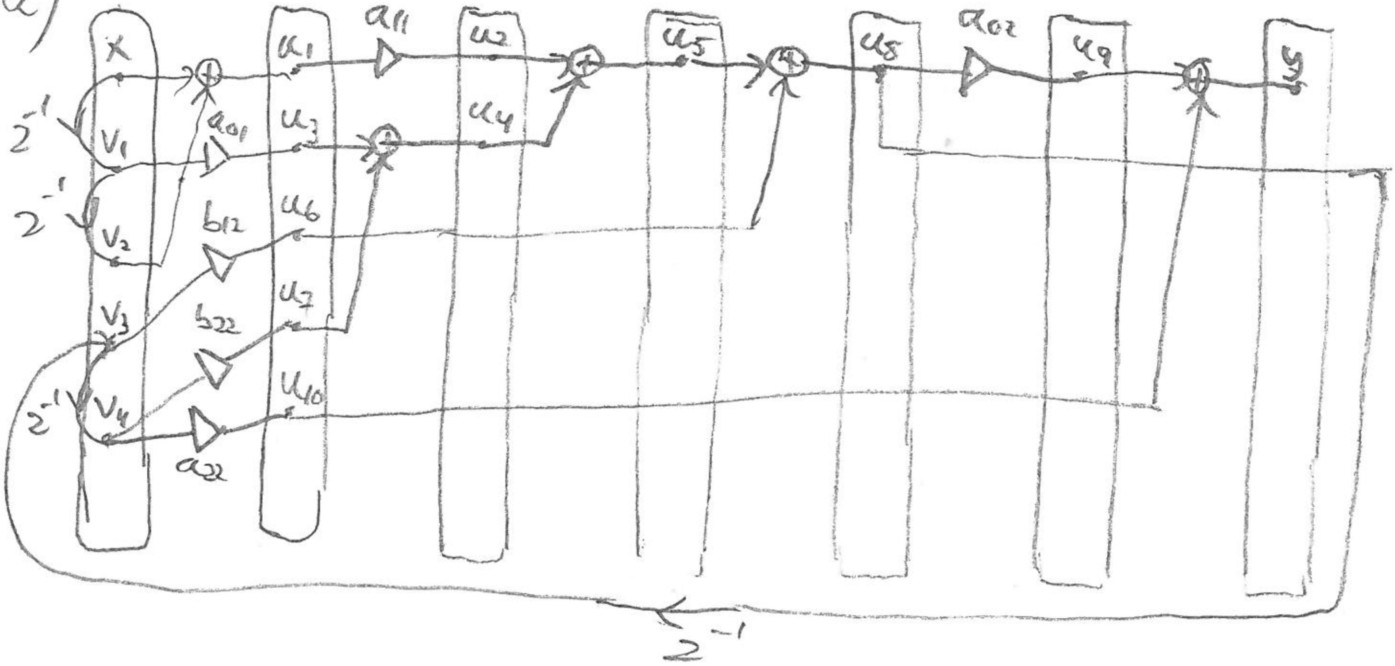
HP: zeros: 4 at $s=0$
poles: $-0,5074 \pm j2.9409$
$-3,6536 \pm j3.6349$
} s-domain

$H_a(s) \to H(z)$, $s = \frac{z-1}{z+1}$, $z = \frac{1+s}{1-s}$ $\left(\frac{2}{T}=1\right)$

zeros: 4 at $z=1$
poles: $-0,7229 \pm j0,5386$
$-0,7331 \pm j0,2084$
} z-domain



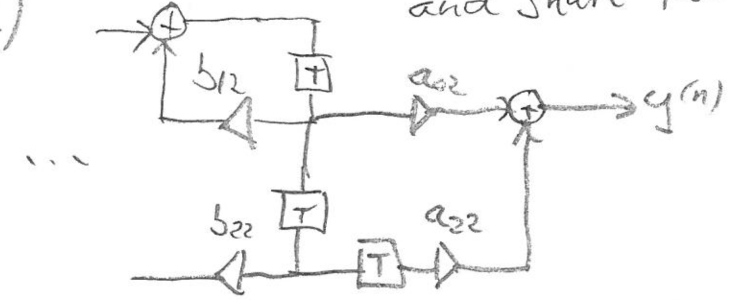
3a)



b) $T_{min} = \max\left\{ \frac{T_{mult} + T_{add}}{1}, \frac{T_{mult} + 3T_{add}}{2} \right\} = T_{mult} + T_{add} = 1{,}25\,\mu s$

$T_{CP} = 2T_{mult} + 4T_{add} = 3\,\mu s$

c) Propagate one delay elem. from the output into the inputs of a_{02} and a_{22} and share two delay elem.

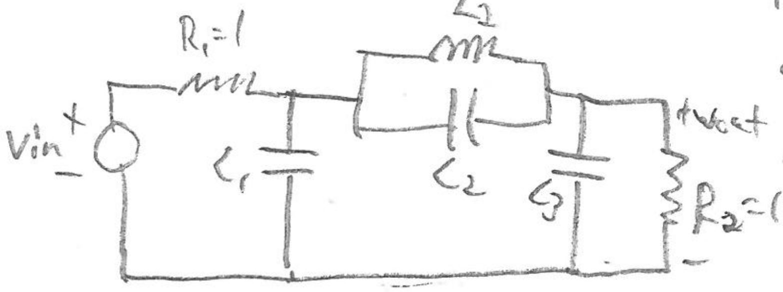


4) $\omega_c T = 2\pi \cdot \frac{f_c}{f_{sample}} = 0{,}3\pi$, $\omega_s T = 2\pi \cdot \frac{f_s}{f_{sample}} = 0{,}75\pi$

$\omega_{ac} = \tan\left(\frac{\omega_c T}{2}\right) = 0{,}5095$, $\omega_{as} = \tan\left(\frac{\omega_s T}{2}\right) = 2{,}4142$

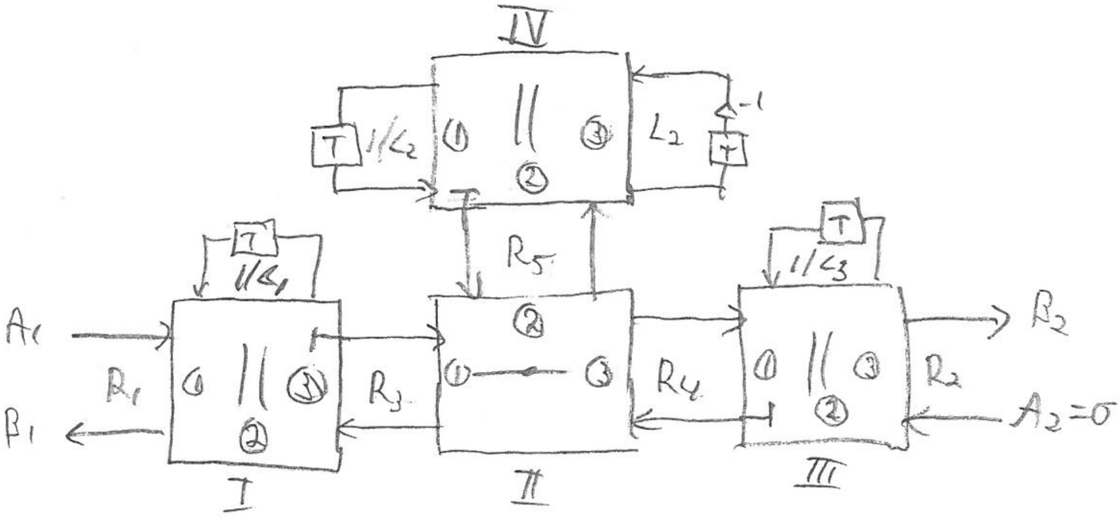
Cauer, $\frac{\omega_{as}}{\omega_{ac}} = 4{,}7382$
$A_{min} = 40, A_{max} = 0{,}1$ $\Rightarrow$ order $N = 3$

Table p. 27, $13 \leq \Theta \leq 16$
choose, e.g., $\Theta = 15$



Normalized: $C_1' = C_3' = 0{,}9944$
$C_2' = 0{,}04631$
$L_2' = 1{,}0941$

Denormalize, divide by ω_{ac} $(R=1)$
$\Rightarrow C_1 = C_3 = 1{,}9516$, $C_2 = 0{,}0909$
$L_2 = 2{,}1473$



I: $G_3 = \frac{1}{R_1} + C_1$

$\alpha_1 = \frac{1/R_1}{1/R_1 + C_1} = 0{,}3388$

$\alpha_2 = 1 - \alpha_1 = 0{,}6612$

$\alpha_3 = 1$

III: symm. with I: $G_4 = G_3$

$\alpha_1 = 1$
$\alpha_2 = 0{,}6612$
$\alpha_3 = 0{,}3388$

IV: $G_5 = \frac{1}{L_2} + C_2 = 0{,}5566$

$\alpha_1 = \frac{2C_2}{C_2 + G_5 + 1/L_2} = 0{,}1633$

$\alpha_2 = 1$, $\alpha_3 = 1 - \alpha_1 = 0{,}8367$

II: $\alpha_1 = \frac{2R_3}{R_3 + R_5 + R_4} = 0{,}2739$ $\left(R = \frac{1}{G}\right)$

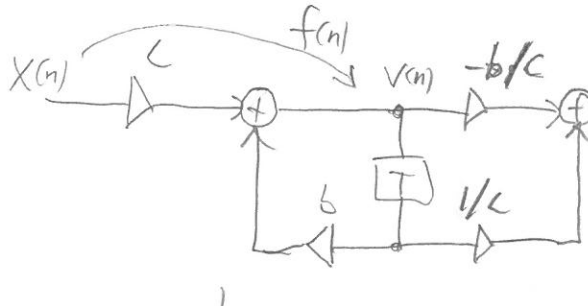
$\alpha_2 = 2 - 2\alpha_1 = 1{,}4523$

$\alpha_3 = \alpha_1$

5a) $P_{ye} = \frac{Q^2}{12} \cdot \Big(\underbrace{\sum_{n=0}^{\infty} h^2(n)}_{=1} + \underbrace{\sum_{n=0}^{\infty} g^2(n)}_{=1} \Big) = \frac{Q^2}{6} = 6{,}21 \cdot 10^{-10}$

$=1$, $g(n) = \delta(n)$ direct propagation from output of $-b$ to y

$=1$
Allpass from output of b to y

b) 

scaled since unity-gain allpass filter from x to y

in L_2-norm sense

$c = \frac{1}{\sqrt{\sum_{n=0}^{\infty} f^2(n)}}$ scales v(n) and thus the inputs to the multipliers

$f(n) = b^n \cdot u(n) \Rightarrow \sum_{n=0}^{\infty} f^2(n) = \frac{1}{1-b^2} = 1{,}1636$

(with $c=1$)

$\Rightarrow c = 0{,}9270$

6a) $\frac{2k\pi}{M} = \pi$ to ensure a second passband centered on π

$\Rightarrow M = 2k$, k integer

b) $\omega_c^G T = M \cdot \omega_{c_1}^H T$, $\omega_s^G T = M \cdot \omega_{s_1}^H T$

$\omega_{c_1}^F T = \omega_{c_1}^H T$, $\omega_{s_1}^F T = \frac{2\pi}{M} - \omega_{s_1}^H T$

$\omega_{c_2}^F T = \pi - \omega_{c_1}^F T$, $\omega_{s_2}^F T = \pi - \omega_{s_1}^F T$

$= \omega_{c_2}^H T$

7) A_0, A_1 : unity-gain allpass $\Rightarrow A_k(e^{j\omega T}) = e^{j\Phi_k(\omega T)}$, $k=0,1$

(ωT) ignored for simplicity

$$\Rightarrow H(e^{j\omega T}) = 0.5\left[e^{j\Phi_0}+e^{j\Phi_1}\right] = e^{j0,5(\Phi_0+\Phi_1)}\cdot\cos\left(\frac{\Phi_0-\Phi_1}{2}\right)$$

$$H_c(e^{j\omega T}) = \quad - \quad = j\,e^{j0,5(\Phi_0+\Phi_1)}\cdot\sin\left(\frac{\Phi_0-\Phi_1}{2}\right)$$

$$\therefore |H|^2+|H_c|^2 = \cos^2(\ )+\sin^2(\ ) = 1$$

b) Due to the power complementary property we have

$$(1-\delta_c)^2+\delta_s^2 = 1 \iff \delta_c \approx 0,5\cdot\delta_s^2 = 1,25\cdot 10^{-5}$$

$\Rightarrow$ order $= 6$ but order of LWDF must be odd for LP & HP filters $\Rightarrow$ <u>order $= 7$</u>

c) $\Rightarrow \tau_g(\omega T) = \dfrac{\tau_0(\omega T)+\tau_1(\omega T)}{2}$ for both H & H_c

(but additional diracs for different ωT due to phase jumps)