LINKÖPING UNIVERSITY

Exam in TSTE06 Digital Filters

Exam code:	TEN1
Date:	2019-05-29 **Time:** 14–18
Place:	TER3
Examiner:	Håkan Johansson
Department:	ISY
Allowed aids:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables
Number of tasks:	7
Grading:	Maximum 70 points, where 30, 42, and 56 points are required for the grades 3, 4, and 5, respectively. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE06/
Result:	Available by 2019-06-13

1 **a.** How are the group delay and phase delay related to the phase response of a filter? (2 p)

b. What does forced-response stability mean? (2 p)

c. What is the difference between recursive and nonrecursive structures? How are they related to FIR and IIR filters? (2 p)

d. Sketch typical magnitude responses for highpass filters designed in the minimax sense and least-squares sense, respectively. (2 p)

e. Indicate in the z-plane typical zero locations for the following digital filters: 5th-order lowpass Cauer filter, 9th-order highpass Chebyshev-II filter, 10th-order bandpass Butterworth filter, 8th-order bandstop Chebyshev-I filter. (2 p)

2 Synthesize a minimum-order Cauer filter that meets the specification below. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. (10 p)

$\omega_c T = 0.13\pi$, $\omega_s T = 0.5\pi$, $A_{\text{max}} = 0.09883$ dB ($\rho = 15\%$), $A_{\text{min}} = 40$ dB.

3 Realize a minimum-order Chebyshev-I T-type ladder wave digital filter that meets the specification below. Assume source and load resistor values $R_1 = R_2 = 1$. Use Richards variable $\Psi = \frac{z-1}{z+1}$ and directly interconnected adaptors with reflection-free ports. Draw the block diagram and compute the adaptor coefficients. (10 p)

$\omega_c T = 0.8\pi$, $\omega_s T = 0.55\pi$, $A_{\text{max}} = 0.5$ dB, $A_{\text{min}} = 50$ dB.

4 A filter structure is given according to Figure 1. It is composed of two first-order unity-gain allpass sections in cascade, where each section is realized using a symmetric two-port adaptor.

a. Draw the signal-flow graph in precedence form. (6 p)

b. Determine the critical path (T_{CP}) and minimal sampling period (T_{min}), expressed in terms of T_{add} and T_{mult}. (2 p)

c. Use pipelining so that the new T_{CP} equals T_{min}. (2 p)

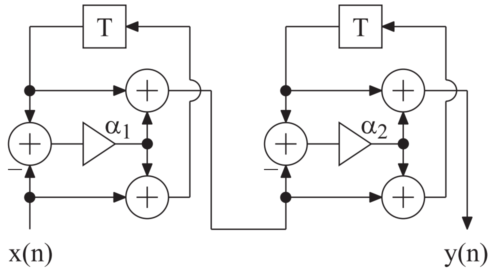

Figure 1:

5 **a.** Compute the variance of the roundoff noise at the output of the filter structure seen in Figure 1 (i.e., the same filter structure as in Problem 4). Assume that $\alpha_1 = 0.25$ and $\alpha_2 = 0.75$. Assume further that quantization (rounding) is carried out after each multiplication, and that the quantizations can be modeled as uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-12}$. Utilize that, when white noise propagates through an LTI system with the impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. Also utilize that the filter is composed of two unity-gain allpass filters in cascade. (5 p)

Noise gain of an LTI system with the impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$\text{Noise Gain} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b. Scale the filter using the L_2-norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. Again, utilize that the filter is composed of two unity-gain allpass filters in cascade. (5 p)

L_2-norm of an LTI system with the impulse response $f(n)$ and frequency response $F(e^{j\omega T})$:

$$||F(e^{j\omega T})||_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

6 A system for interpolation by $L = L_1L_2 = 18$ is to be realized in two steps according to Fig. 2(a).

a. Express the overall transfer function $H(z)$ (as a function of H_1 and H_2) in the mathematically equivalent one-stage system in Fig. 2(b) which interpolates by L through upsampling by L followed by the filter $H(z)$. (2 p)

b. Use the filter order estimate given below to estimate the values of L_1 and L_2 that give the smallest overall number of multiplications per output sample for the scheme in Fig. 2(a). Assume for simplicity that the effect of the passband and stopband ripples can be ignored (i.e., assume that both filters have the same ripples) and that the passband and stopband edges of the overall filter $H(z)$ should be $\pi/L \pm \Delta$ where $\Delta = 0.075\pi/L$. For the transition band (and thus order estimation) of $H_2(z)$, the effect of Δ can be ignored. Also assume that polyphase interpolator structures are used, so that for each stage, the filtering is carried out at its input sampling rate. (6 p)

c. Based on the estimates in b. above, select suitable values of L_1 and L_2. (2 p)

Filter order estimate for a filter with passband ripple δ_c, stopband ripple δ_s, passband edge $\omega_c T$, and stopband edge $\omega_s T$:

$$N \approx \frac{K}{\omega_s T - \omega_c T}$$

where

$$K = -\frac{4\pi}{3}\log_{10}(13\delta_c\delta_s)$$

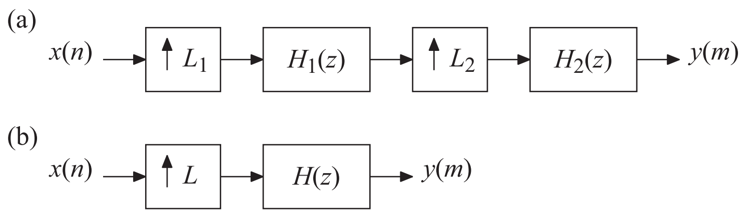

Figure 2:

7 In a four-fold polyphase representation of a transfer function $H(z)$, it can be expressed as

$$H(z) = H_0(z^4) + z^{-1}H_1(z^4) + +z^{-2}H_2(z^4) + +z^{-3}H_3(z^4)$$

Apply an appropriate number of pole-zero cancellations to the transfer function $H(z)$ below, and derive the polyphase components $H_k(z)$, $k = 0, 1, 2, 3$ in the polyphase representation above. This technique is referred to as scattered-look-ahead and can be used to increase the maximal sampling frequency of IIR filters. This is because, after applying the pole-zero cancellations, the denominator (recursive part) will be a function of z^{-4} (z^{-M} in general) which corresponds to 4 delay elements in series in an implementation, implying an increase of the maximal sampling frequency by a factor of 4. (10 p)

$$H(z) = \frac{1 - z^{-1}}{1 - bz^{-1}}, \quad |b| < 1$$