LINKÖPING UNIVERSITY

Exam in TSTE06 Digital Filters

Exam code:	TEN1
Date:	2018-05-30 **Time:** 14–18
Place:	TER3
Examiner:	Håkan Johansson
Department:	ISY
Allowed aids:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables
Number of tasks:	7
Grading:	Maximum 70 points, where 30, 42, and 56 points are required for the grades 3, 4, and 5, respectively. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE06/
Result:	Available by 2018-06-13

1 **a.** Explain what high sensitivity of a filter structure means and implies. (2 p)

b. Which are the critical nodes that need to be scaled in digital filters when two's-complement arithmetic is used? What is the difference between safe scaling and L_p-norm scaling? (2 p)

c. Why must some ports be reflection-free when realizing wave digital filters using directly interconnected adaptors? (2 p)

d. An FIR filter transfer function is given as: $H(z) = z^{-1} + 2z^{-3} + z^{-5}$. Does the filter have a linear phase response? (2 p)

e. Indicate in the z-plane typical zero locations for the following digital filters: 7th-order highpass Cauer filter, 5th-order lowpass Chebyshev-II filter, 8th-order bandpass Chebyshev-I filter, 10th-order bandstop Butterworth filter.

(2 p)

2 Synthesize a digital Chebyshev-I filter that meets the attenuation specification in Figure 1. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. (10 p)

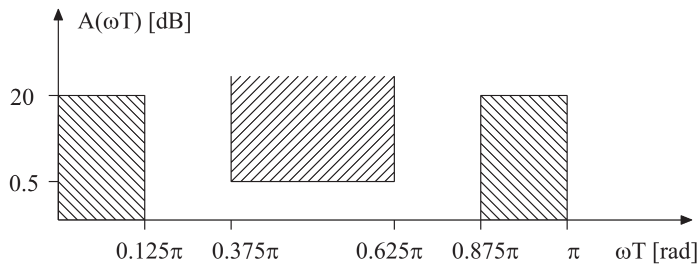

Figure 1:

3 A filter structure is given according to Figure 2.

a. Draw the signal-flow graph in precedence form. (6 p)

b. Determine the minimal sampling period, T_{min}, in terms of T_{mult} and T_{add}. (1 p)

c. Use pipelining so that the critical path, T_{CP}, of the pipelined structure contains one multiplier and two adders. The pipelined structure should not have more than four delay elements in total. (3 p)

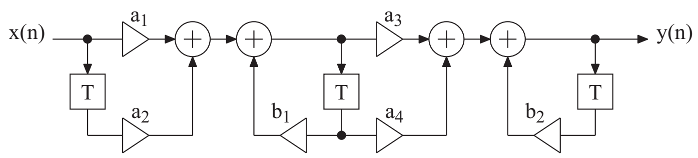

Figure 2:

4 **a.** The filter in Figure 3 corresponds to a first-order so called allpole section in cascade with a first-order allpass section. Compute the variance of the roundoff noise at the output of the filter when $a = 0.375$ and $b = -0.625$. Assume that quantization (rounding) is carried out after each multiplication, and that the quantizations can be modelled as uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-12}$. Utilize that, when white noise propagates through an LTI-system with the impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain of an LTI system with the impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$\text{Noise Gain} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b. Scale the filter when $a = 0.375$ and $b = -0.625$, using the L_2-norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. (5 p)

L_2-norm of an LTI system with the impulse response $f(n)$ and frequency response $F(e^{j\omega T})$:

$$||F(e^{j\omega T})||_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

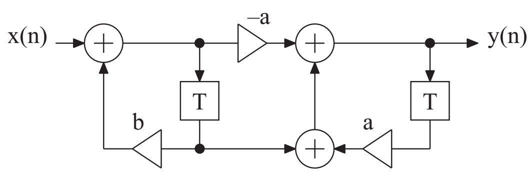

Figure 3:

5 Realize a third-order Cauer wave digital filter satisfying the highpass filter requirements below. Start from a lowpass filter that is realized with a T-net with load and generator resistances of 1 Ω. Use Richards variable $\Psi = (z-1)/(z+1)$ and directly interconnected adaptors with reflection-free ports. Draw the wave-flow graph (block diagram) and compute the adaptor coefficients. (10 p)

$\omega_c T = 0.8\pi$, $\omega_s T = 0.3\pi$, $A_{\max} = 0.1$ dB ($\rho = 15\%$), $A_{\min} = 50$ dB.

6 A digital narrow-band bandpass filter $H(z)$ meeting the requirements given below is to be realized.

$$1 - 0.01 \le |H(e^{j\omega T})| \le 1 + 0.01, \quad \omega T \in [0.24\pi, 0.26\pi]$$
$$|H(e^{j\omega T})| \le 0.001, \quad \omega T \in [0, 0.23\pi] \cup [0.27\pi, \pi]$$

a. Assume that the filter is realized in cascade form according to $H(z) = G(z^M)F(z)$ (frequency-response masking technique), where $G(z)$ is a lowpass filter and $F(z)$ is a bandpass filter. Determine the specifications for $G(z)$ and $F(z)$ when $M = 16$, so that the specification of $H(z)$ is satisfied. The transition bands of $G(z)$ and $F(z)$ should not be narrower than necessary. Motivate the answer by sketching the magnitude responses of the filters involved. (5 p)

b. Assume that the filters $G(z)$ and $F(z)$ are realized using direct-form linear-phase FIR filters of even orders. Estimate the overall computational complexity (number of multiplications and additions) for the filter $H(z)$ above and compare it with the complexity when $H(z)$ is realized directly using only one FIR filter. The order of a linear-phase FIR filter can be estimated with the formula below. For filters with several transition bands, the order is essentially determined by the most narrow of these bands. (5 p)

$$\text{Filter order} \approx \frac{-4\pi \times \log_{10}(13 \times \text{passband ripple} \times \text{stopband ripple})}{3 \times \text{transition band width}}$$

7 The normal-output transfer function $H(z)$ of a lattice WDF, and its complementary-output transfer function $H_c(z)$, are given by $H(z) = 0.5[A_0(z) + A_1(z)]$ and $H_c(z) = 0.5[A_0(z) - A_1(z)]$, where $A_0(z)$ and $A_1(z)$ are unity-gain (magnitude = 1) allpass transfer functions.

a. Show that $H(z)$ and $H_c(z)$ form a power-complementary filter pair, i.e., $|H(e^{j\omega T})|^2 + |H_c(e^{j\omega T})|^2 = 1$. (4 p)

b. Show that the group delay for both $H(z)$ and $H_c(z)$ can be written as $\tau_g(\omega T) = 0.5[\tau_{g0}(\omega T) + \tau_{g1}(\omega T)]$ where $\tau_{g0}(\omega T)$ and $\tau_{g1}(\omega T)$ denote the group delays of $A_0(z)$ and $A_1(z)$, respectively. It is assumed here that we ignore Dirac impulses in $\tau_g(\omega T)$ caused by discontinuities at the zeros of $H(z)$ and $H_c(z)$. (3 p)

c. Assume that $H(z)$ and $H_c(z)$ form a lowpass and highpass filter pair. Assume next that z is replaced with $-z^2$ in $H(z)$ and $H_c(z)$. Which types of filters (lowpass, highpass, bandpass, or bandstop) do $H(-z^2)$ and $H_c(-z^2)$ correspond to? (3 p)