LINKÖPING UNIVERSITY

Exam in TSTE06 Digital Filters

Exam code:	TEN1		
Date:	2016-06-01	**Time:**	14–18
Place:	T1		
Examiner:	Håkan Johansson		
Department:	ISY		
Allowed aids:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables		
Number of tasks:	7		
Grading:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.		
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE06/		
Result:	Available by 2016-06-15		

1

a. Explain what the zero-phase frequency response is for a linear-phase FIR filter. How is it related to the magnitude response? (2 p)

b. In terms of sensitivity, what is the difference between ladder wave digital filters and lattice wave digital filters? (2 p)

c. What is the difference between recursive and nonrecursive structures? How are they related to FIR and IIR filters? (2 p)

d. Sketch typical magnitude responses for lowpass filters designed in the minimax sense and least-squares sense, respectively. (2 p)

e. Draw a block diagram of a system, using upsampler(s), downsampler(s) and filter(s), that increases the sampling rate by a factor of 3.5. Also sketch the ideal frequency response of the filter(s). (2 p)

2 Synthesize a third-order digital high-pass Cauer filter that meets the specification below. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. (10 p)

$\omega_c T = 0.8\pi$, $\omega_s T = 0.38\pi$, $A_{\max} = 0.09883$ dB ($\rho = 15\%$), $A_{\min} = 45$ dB.

3 Realize a minimum-order Chebyshev-I π-type ladder wave digital filter that meets the specification below. Use Richard's variable $\Psi = \frac{z-1}{z+1}$ and directly interconnected adaptors with reflection-free ports. Draw the block diagram and compute the adaptor coefficients. (10 p)

$\omega_c T = 0.2\pi$, $\omega_s T = 0.45\pi$, $A_{\max} = 0.1$ dB, $A_{\min} = 40$ dB.

4 A filter structure is given according to Figure 1.

a. Draw the signal-flow graph in precedence form. (6 p)

b. Use pipelining so that the critical path (T_{CP}) equals the minimal sampling period (T_{min}) of the pipelined structure. The pipelined structure should not have more than three delay elements in total. (4 p)

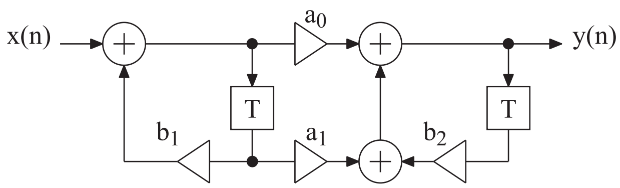

Figure 1:

5 **a.** Compute the variance of the roundoff noise at the output of the filter seen in Figure 2. Assume that quantization (rounding) is carried out after each multiplication, and that the quantizations can be modelled as uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-10}$. Utilize that, when white noise propagates through an LTI-system with the impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain of an LTI system with the impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$\text{Noise Gain} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b. Scale the filter using the L_2-norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. Utilize that Branch 2 of the filter realizes a unity-gain allpass filter. (5 p)

L_2-norm of an LTI system with the impulse response $f(n)$ and frequency response $F(e^{j\omega T})$:

$$||F(e^{j\omega T})||_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

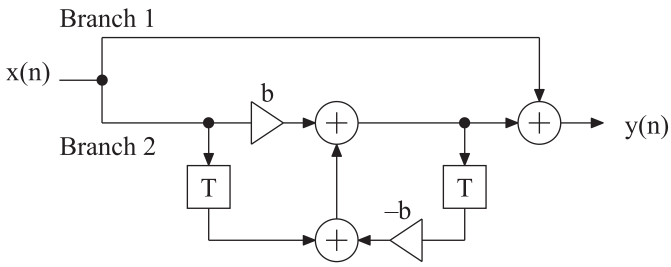

Figure 2:

6 A system for interpolation by $L = L_1 L_2 = 24$ is to be realized through two cascaded stages. The first stage interpolates by L_1 through upsampling by L_1 followed by the filter $H_1(z)$. The output of the first stage is the input of the second stage which interpolates by L_2 through upsampling by L_2 followed by the filter $H_2(z)$.

a. Express the overall transfer function $H(z)$ (as a function of H_1 and H_2) in the mathematically equivalent one-stage system which interpolates by L through upsampling by L followed by the filter $H(z)$. (2 p)

b. Use the filter order estimate given below to estimate the values of L_1 and L_2 that give the smallest overall number of multiplications per output sample. Assume for simplicity that the effect of the passband and stopband ripples can be ignored (i.e., assume that both filters have the same ripples) and that the passband and stopband edges of the overall filter $H(z)$ should be $\pi/L \pm \Delta$ where $\Delta = 0.1\pi/L$. Also assume that polyphase interpolator structures are used, so that for each stage, the filtering is carried out at its input sampling rate. (6 p)

c. Based on the estimates in b. above, select suitable values of L_1 and L_2. (2 p)

Filter order estimate for a filter with passband ripple δ_c, stopband ripple δ_s, passband edge $\omega_c T$, and stopband edge $\omega_s T$:

$$N \approx \frac{K}{\omega_s T - \omega_c T}$$

where

$$K = -\frac{4\pi}{3}\log_{10}(13\delta_c\delta_s)$$

7 Assume that the digital filter transfer function $H(z)$ is obtained from the analog filter transfer function $H_a(s)$ using the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$.

a. Show that $H(z)$ is stable (unstable) when $H_a(s)$ is stable (unstable). (4 p)

b. Show that $s = j\omega_a$ corresponds to $z = e^{j\omega T}$ where $\omega_a = \frac{2}{T}\tan(\omega T/2)$. (3 p)

c. Assume that the group delay of $H_a(s)$ is constant (frequency independent). Show that the group delay of $H(z)$ is not constant. (3 p)