Digital Filters 190529

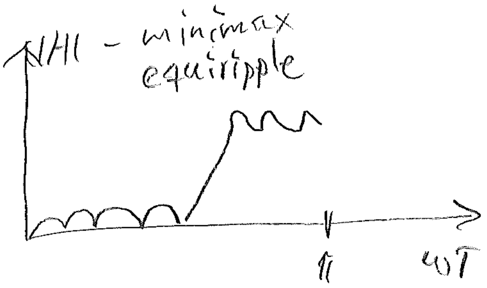
1 a) Group delay: $\tau_g(\omega T) = -\frac{\partial \phi(\omega T)}{\partial \omega}$ $\left(\text{or } -\frac{\partial \phi(\omega T)}{\partial (\omega T)}\right)$

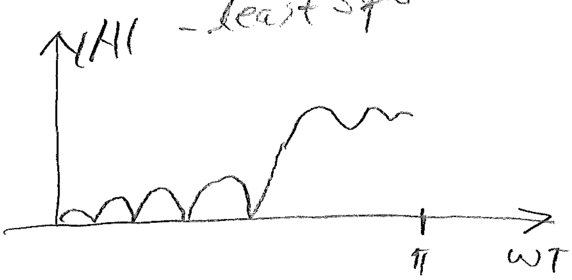
Phase delay: $\tau_p(\omega T) = -\frac{\phi(\omega T)}{\omega}$ $\left(\text{or } -\frac{\phi(\omega T)}{\omega T}\right)$

b) Stable filter under finite arithmetic conditions
Recovers after disturbance, no parasitic oscillations

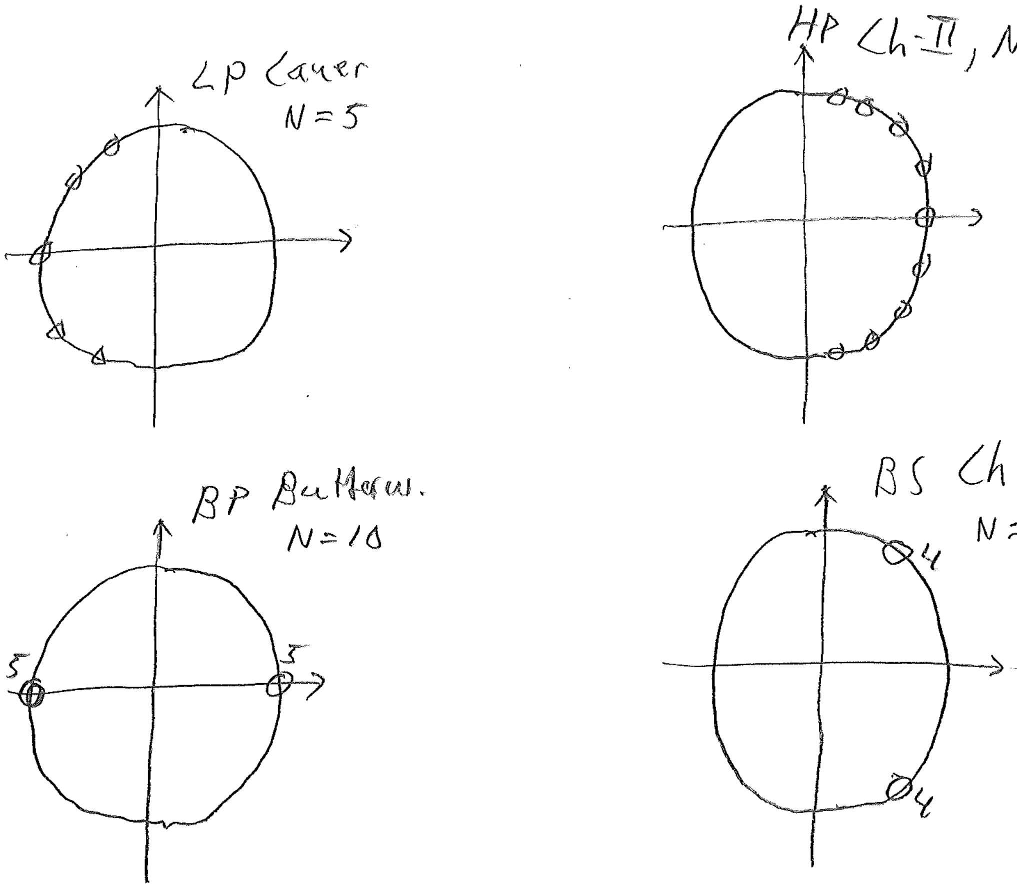
c) Recursive - output computed using input samples and previous output samples
Nonrecursive - output computed using only input samples
IIR always recursive
FIR normally nonrecursive, can be recursive

d)





e)



2) Edges of analog filter ($\frac{2}{T}=1$ for simplicity)

$w_{ac} = \tan\left(\frac{w_c T}{2}\right) = 0{,}207 \qquad w_{as} = \tan\left(\frac{w_s T}{2}\right) = 1$

$\frac{w_{as}}{w_{ac}} = 4{,}82 \Rightarrow$

$12 \le \theta \le 16$ select $\theta = 14$

Filter order $N = 3$

Normalized zeros: $\pm j4.7352$, one at $s=\infty$

poles: -1.0612

$-0.4633 \pm j1.2119$

Denormalized zeros $\pm j0{,}9848$, one at $s=\infty$

Mult. by w_{ac} poles $-0{,}2073$

$-0{,}09594 \pm j0{,}2510$

$s \to z = \frac{1+s}{1-s}$:

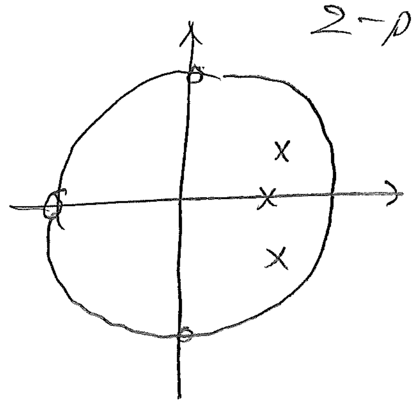
zeros: $0{,}01536 \pm j0{,}9999$

$-1{,}0000$

poles: $0{,}6565$

$0{,}7340 \pm j0{,}3971$

z-plane



3) Edges of analog reference filter

$\omega_{arc} = \tan\left(\frac{0.8\pi}{2}\right) = 3.078$, $\omega_{ars} = \tan\left(\frac{0.55\pi}{2}\right) = 1.171$

HP → LP spec. $\Omega_{arc} = \frac{\omega_T^2}{\omega_{arc}} = \left/ \text{select } \omega_T^2 = \omega_{arc} \right/ = 1$

$$\Omega_{ars} = \frac{\omega_T^2}{\omega_{ars}} = \frac{\omega_{arc}}{\omega_{ars}} \approx 2.629$$

$\frac{\Omega_{ars}}{\Omega_{arc}} = 2{,}629 \Rightarrow$ Filter order $N = 5$

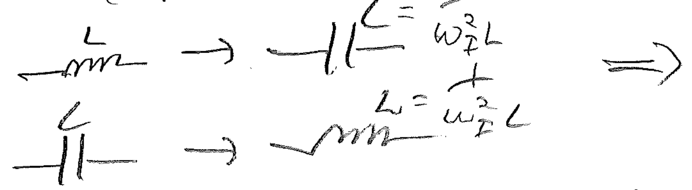
Normalized elem. values $L_1' = L_5' = 1.7058$, $L_3' = 2.5408$

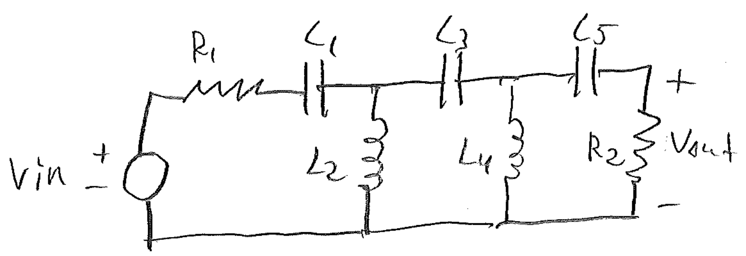
$C_2' = C_4' = 1.2296$

Denormalize with $R = 1$, $\Omega_{arc} = 1 \Rightarrow L = L'$, $C = C'$

$(= R_1 = R_2 = 1)$

LP → HP :



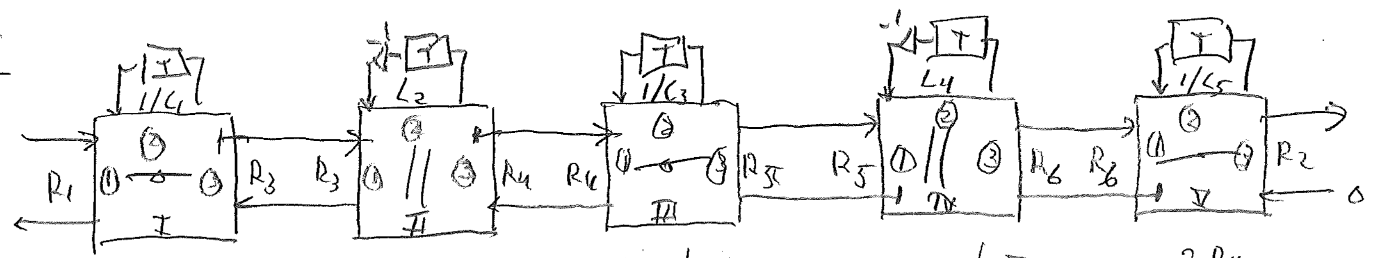


$L_2 = L_4 = \frac{1}{\omega_T^2 C_2} = 0{,}2642$

$C_1 = C_5 = \frac{1}{\omega_T^2 L_1} = 0{,}1905$

$C_3 = \frac{1}{\omega_T^2 L_3} = 0{,}1279$, $R_1 = R_2 = 1$

WDF



I: $R_3 = R_1 + \frac{1}{C_1} = 6{,}2499$

$\alpha_1 = \frac{2R_1}{R_1 + 1/C_2 + R_3} = 0{,}1600$

$\alpha_2 = 1 - \alpha_1 = 0.8400$

$\alpha_3 = 1$

II : $R_4 = \frac{1}{1/R_3 + 1/L_2} = 0{,}2535$

$\alpha_1 = \frac{2/R_3}{1/R_3 + 1/L_2 + 1/R_4} = 0{,}04057$

$\alpha_2 = 1 - \alpha_1 = 0{,}95943$

$\alpha_3 = 1$

III: $\alpha_1 = \frac{2R_4}{R_4 + 1/C_3 + R_5} = 0{,}06089$

$\alpha_2 = 2 - \alpha_1 - \alpha_3 = 1.8782$

$\alpha_3 = \alpha_1 = 0{,}06089$

$R_4 = R_5$

IV : $R_5 = R_4$, $\alpha_1 = 1$

$\alpha_2 = 0.95943$

$\alpha_3 = 0{,}04057$

(Symm. w. II)

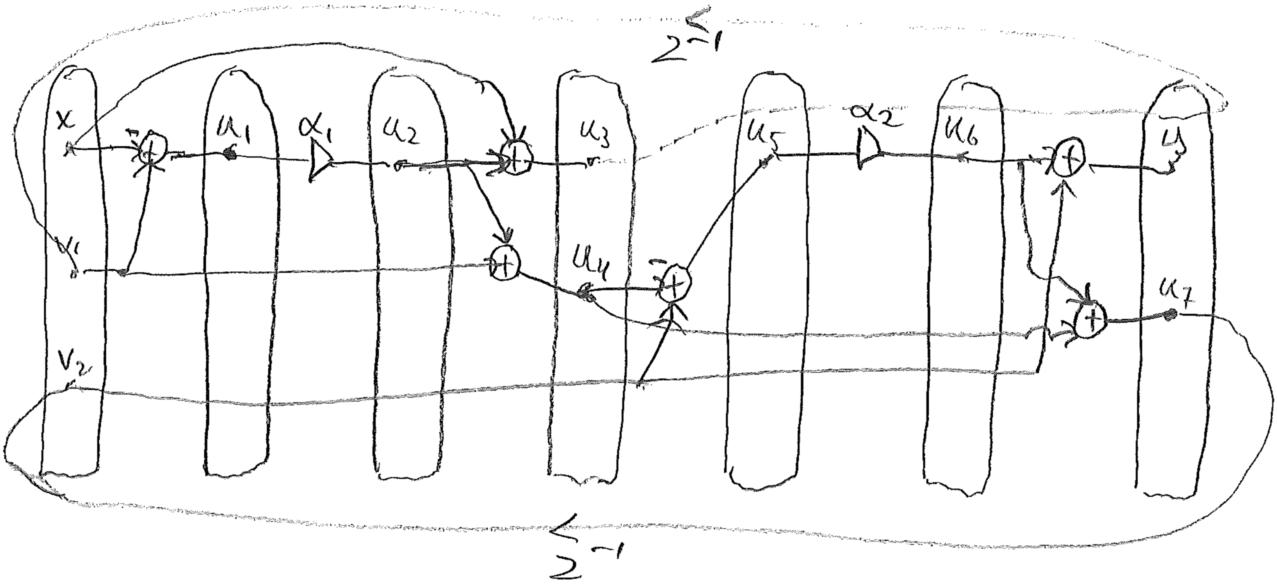
V: $R_6 = R_3$, $\alpha_1 = 1$

$\alpha_2 = 0{,}7520$

$\alpha_3 = 0{,}2480$

(Symmetry with I)

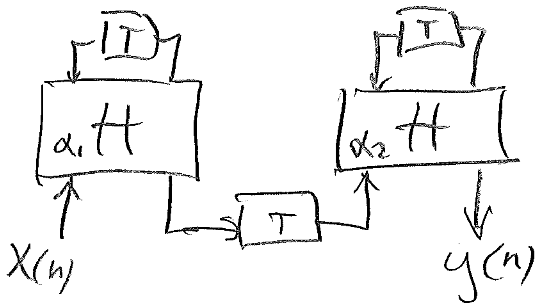
4a)



b) $T_{CP} = 4T_{add} + 2T_{mult}$

$T_{min} = 2T_{add} + T_{mult}$

c)



$\Rightarrow T_{CP} = 2T_{add} + T_{mult} = T_{min}$

5a) $\frac{N}{2}+1$ uncorrelated noise sources, propagate directly to the output ($g(n) = \delta(n)$)

$\Rightarrow$ Noise variance (average power) $P_{ye} = \left(\frac{N}{2}+1\right) \cdot \frac{Q^2}{12}$

$= \left(\frac{20}{2}+1\right) \cdot \frac{(2^{-12})^2}{12} =$

$\approx 5.464 \times 10^{-8}$

b) Inputs to mult. scaled by mult. the input $x(n)$ by $c_1 = \frac{1}{2}$ $\left(c_1 = \frac{1}{\sum_n |f_i(n)|} = \frac{1}{2}\right)$, $f_i(n) = \delta(n) + \delta(n-1)$

Impulse response from $x(n)$ to the input of mult $h(i)$, $i = 0, \ldots \frac{N}{2}$)

Output $y(n)$ scaled by mult. the original mult $h(n)$

by $c_2 = \frac{1}{c_1 \cdot \sum_{n=0}^{N} |h(n)|} = \frac{1/c_1}{|h(\frac{N}{2})| + 2 \cdot \sum_{n=0}^{N/2-1} |h(n)|}$

Due to scaling above

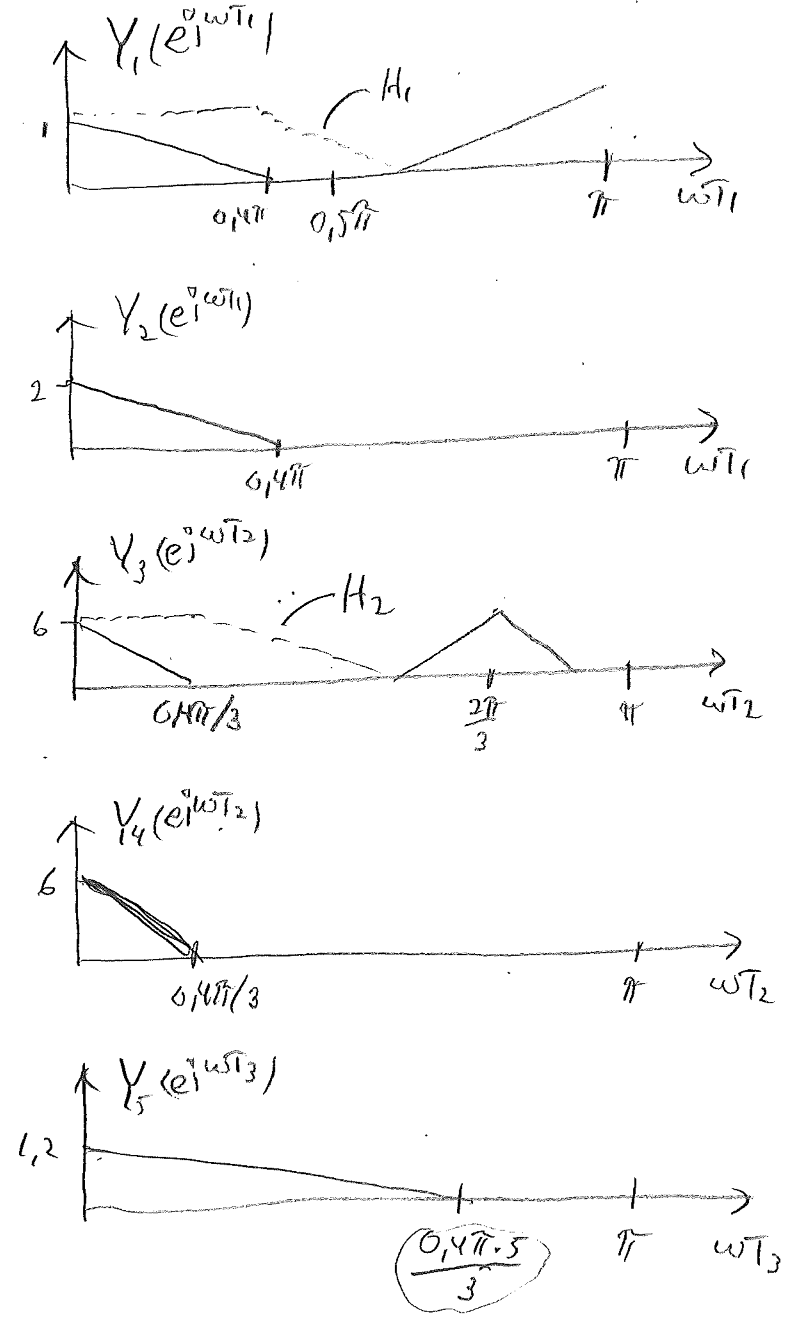
$= \frac{2}{11 + 2\sum_{n=0}^{9}(n+1)} = \frac{2}{11 + 2 \cdot 55} = \frac{2}{121} \approx 0{,}01653$

6a) Interp. factor $= 2 \times 3/5 = 1.2$

b) H_1: Edges $0{,}4\pi$, $0{,}6\pi$, Gain $= 2$

H_2: Edges $\frac{0{,}4\pi}{3}$, $\frac{2\pi}{3} - \frac{0{,}4\pi}{3} = \frac{1.6\pi}{3}$, Gain $= 3$

c)



$= \frac{2\pi}{3} \approx 0{,}667\pi$

7a) For all structures: N delay elements required

Direct form: Type I, II, IV : $N+1$ mult

III : $N-1$ mult

Linear-phase direct form: Type I: $\frac{N}{2}+1$ mult

II, IV : $\frac{N+1}{2}$ mult

III : $\frac{N}{2}$ mult

b) Type III & IV: $H(e^{j\omega T}) = j \cdot e^{-j\omega T \cdot N/2} \cdot H_R(\omega T)$

Cascade of III & IV $\Rightarrow$ (since $j \cdot j = -1$)

$$H(e^{j\omega T}) = e^{-j\omega T(N_3+N_4)/2} H_R(\omega T)$$

where $H_R(\omega T) = -H_R^{(3)}(\omega T) \cdot H_R^{(4)}(\omega T)$

$\therefore$ The result is a Type II filter due to and N_3+N_4 is odd when N_3 is even and N_4 odd