Solutions to Exam in Digital Filters 180530

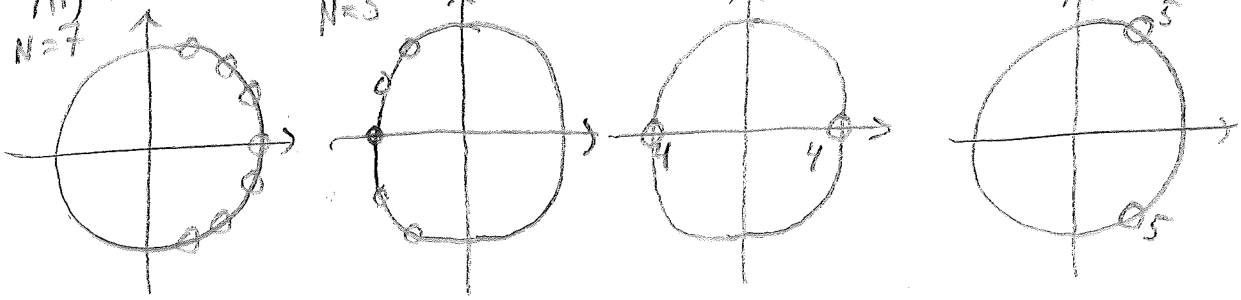
1a) High sensitivity - larger variations in $H(z)$ when coefficients vary $\Rightarrow$ longer coeff. wordlength

b) Inputs to noninteger mult., and output are the critical nodes
safe scaling - never overflow
L_p-norm scaling - overflows with certain probability

c) To avoid delay-free loops

d) Symmetric impuls response around $n=3$ $\Rightarrow$ Linear phase

e) HP, Cauer $N=7$ · LP, Ch-II $N=5$ · BP, Ch-I $N=8$ · BS, Butt.w. $N=10$



2) Edges of analog BP filter (use $\frac{2}{T}=1$)

$$\omega_{ac1} = \tan\left(\frac{0{,}375\pi}{2}\right) = 0{,}6682, \quad \omega_{ac2} = \tan\left(\frac{0{,}625\pi}{2}\right) = 1{,}4966$$

$$\omega_{as1} = \tan\left(\frac{0{,}125\pi}{2}\right) = 0{,}1989, \quad \omega_{as2} = \tan\left(\frac{0{,}875\pi}{2}\right) = 5{,}0273$$

BP $\to$ LP: $\omega_{ac1}\omega_{ac2} \overset{\text{OK!}}{=} \omega_{as1}\omega_{as2} = \omega_I^2 = 1$

$$\left.\begin{aligned} \Omega_{ac} &= \omega_{ac2} - \omega_{ac1} = 0{,}8284 \\ \Omega_{as} &= \omega_{as2} - \omega_{as1} = 4{,}8284 \end{aligned}\right\} \Rightarrow \frac{\Omega_{as}}{\Omega_{ac}} = 5{,}83$$

Filter order $N=2$

Normalized poles $p'_{aLP} = -0{,}7128 \pm j1{,}0040$

zeros Two at $s=\infty$

Denormalize (Mult. by Ω_{ac}) $\Rightarrow$

Denormalized poles: $P_{alp} = -0{,}5905 \pm j\,0.8315$

zeros: Two at $\hat{s}=\infty$

<u>LP → BP</u>: $\hat{s} = s + \frac{\omega_T^2}{s} = \left(\omega_T^2 = 1\right) = s + \frac{1}{s} = \frac{s^2+1}{s}$

$\Leftrightarrow s^2 - \hat{s}\cdot s + 1 = 0 \Leftrightarrow$

$s = \frac{\hat{s}}{2} \pm \sqrt{\left(\frac{\hat{s}}{2}\right)^2 - 1} \Rightarrow$

poles: $-0.1782 \pm j\,0.6327$

$-0.4124 \pm j\,1.4645$

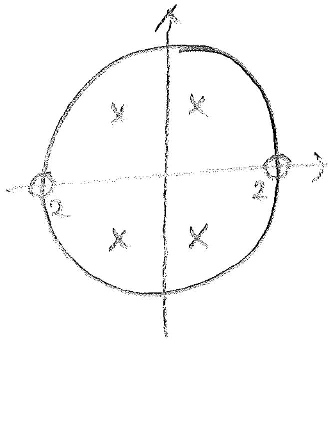
zeros: Two at $s=\infty$, Two at $s=0$

<u>$s \to z$</u> $\quad z = \frac{1+s}{1-s} \quad \left(\frac{2}{T} = 1\right) \Rightarrow$

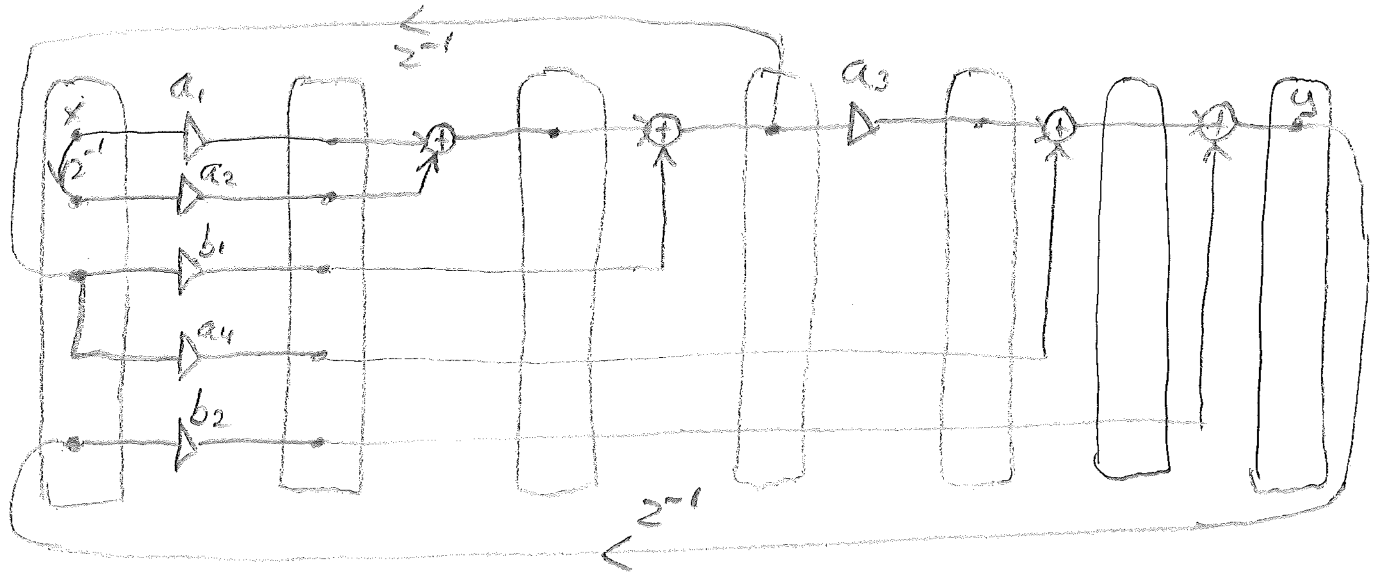
poles: $-0.3176 \pm j\,0.7076$

$0.3176 \pm j\,0.7076$

zeros: Two at $z=1$, Two at $z=-1$



3a)



b) $T_{min} = T_{mult} + T_{add}$ (Two loops, each with one mult, and one add.)

c)

4a) From output of a-mult $\to y$: $h_1(n) = \delta(n) \Rightarrow \sum_{n=0}^{\infty} h_1^2(n) = 1$

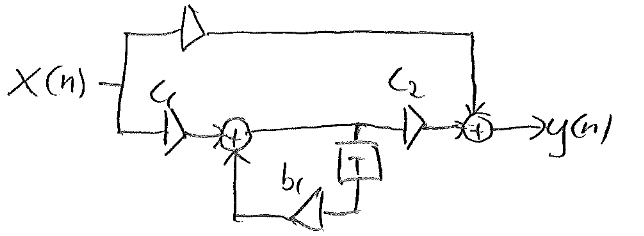
$\Rightarrow P_{e_1} = 1 \cdot P_e = 1 \cdot \frac{Q^2}{12}$

From output of b-mult $\to y$: $h_2(n) = b^n \cdot u(n) \Rightarrow \sum_{n=0}^{\infty} h_2^2(n) = \frac{1}{1-b^2}$

$\Rightarrow P_{e_2} = \frac{1}{1-b^2} \cdot P_e$

In total: $P_e = P_{e_1} + P_{e_2} = \left(1 + \frac{1}{1-b^2}\right) \cdot P_e = 2{,}16 \cdot \frac{2^{-24}}{12} = 1.07 \cdot 10^{-8}$

b) Scaled structure



c_1: $f_1(n) = h_2(n) \Rightarrow \sum_{n=0}^{\infty} f_1^2(n) = \frac{1}{1-b^2} = 1{,}1636$
(with $c_1 = 1$)

$c_1 = \frac{1}{\sqrt{1{,}16}} = 0.9270$

c_2: $f_2(n) = a \cdot \delta(n) + f_1(n) \Rightarrow$
(with $c_1 = c_2 = 1$)

$\sum_{n=0}^{\infty} f_2^2(n) = (1+a)^2 + \sum_{n=1}^{\infty} (b^2)^n = (1+a)^2 + \frac{b^2}{1-b^2} =$

$= 3{,}6793 \Rightarrow$

$c_2 = \frac{1}{\sqrt{3{,}6793}} \cdot \frac{1}{c_1} = 0.5624$

5) Spec. for analog ref. filter

$\omega_{arc} = \tan\left(\frac{0{,}8\pi}{2}\right) = 3{,}0777$, $\omega_{ars} = \tan\left(\frac{0{,}3\pi}{2}\right) = 0{,}5095$

HP → LP spec.: $\Omega_{ac} = \frac{\omega_I^2}{\omega_{arc}} = /\text{select } \omega_I^2 = \omega_{arc}/ = 1$

$$\Omega_{as} = \frac{\omega_I^2}{\omega_{ars}} = \frac{\omega_{arc}}{\omega_{ars}} = 6.0406$$

T&F p. 27, $N=3$, $\rho=15\%$, requirements on $\Omega_{as}/\Omega_{ac} = \Omega_{as} \le 6{,}04$
and $A_{min} \geq 50$

$\Rightarrow$ $10 \le \Theta \le 11$ select $\Theta = 10$ $\Rightarrow$

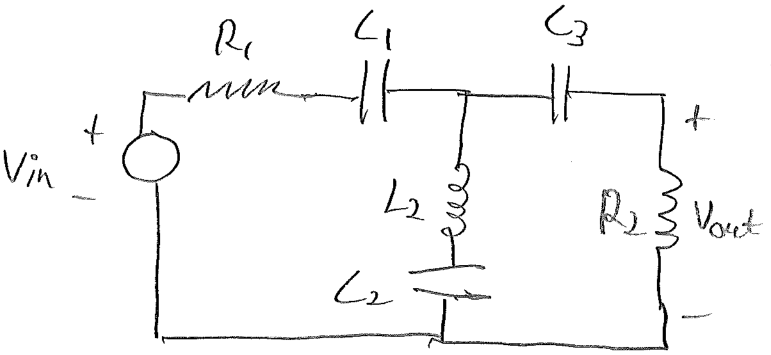
Normalized elementvalues $L_1' = L_3' = 1.0134$, $L_2' = 0{,}0202$

$C_2' = 1.1235$

Denormalize with $R=1$ and $\Omega_{ac} = 1$ $\Rightarrow$ $L = L'$, $C = C'$

LP → HP: $L \rightarrow C$, $C = \frac{1}{\omega_I^2 L}$

$C \rightarrow L$, $L = \frac{1}{\omega_I^2 C}$ $\Big\} \Rightarrow$



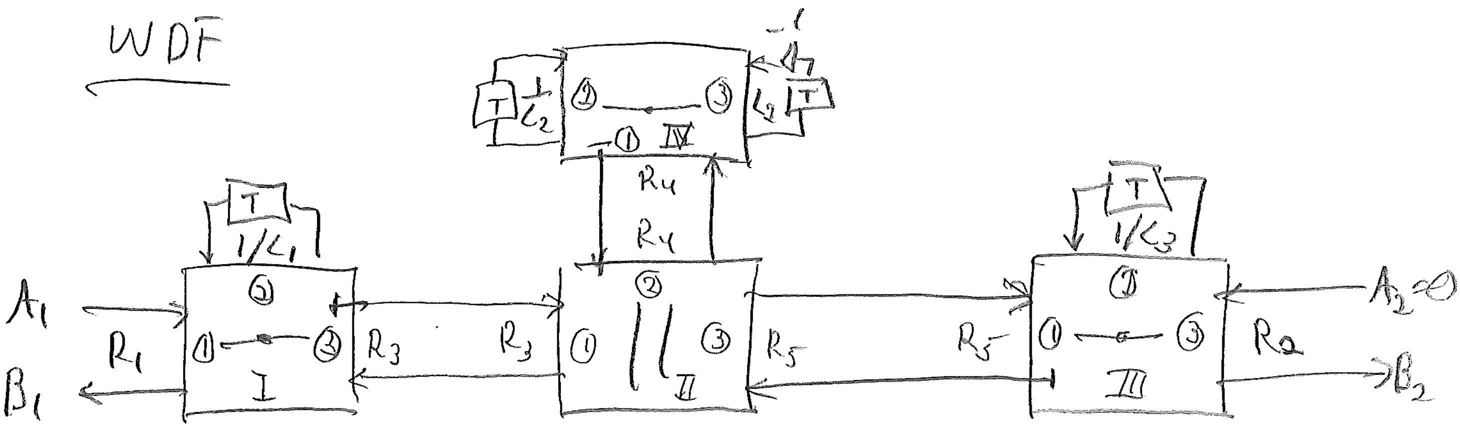
$C_1 = C_3 = \frac{1}{\omega_I^2 L_1} = 0.3206$

$C_2 = \frac{1}{\omega_I^2 L_2} = 16.0850$

$L_2 = \frac{1}{\omega_I^2 C_2} = 0.2892$

$R_1 = R_2 = 1$

WDF



I: $R_3 = R_1 + \frac{1}{C_1} = 4.1192$

$\alpha_1 = \frac{2R_1}{R_1 + 1/C_1 + R_3} = 0.2428$

$\alpha_2 = 2 - \alpha_1 - \alpha_3 = 1 - \alpha_1 = 0.7572$

$\alpha_3 = 1$

III: symmetry with I $\Rightarrow$

$R_5 = R_3 = 4.1192$

$\alpha_1 = 1$

$\alpha_2 = 0.7572$

$\alpha_3 = 0.2428$

IV: $R_4 = L_2 + \frac{1}{C_2} = 0.3514$

$\alpha_1 = 1$

$\alpha_2 = \frac{2L_2}{R_4 + L_2 + \frac{1}{C_2}} = \frac{L_2}{R_4} = 0.8230$

$\alpha_3 = 2 - \alpha_1 - \alpha_2 = 0.1770$

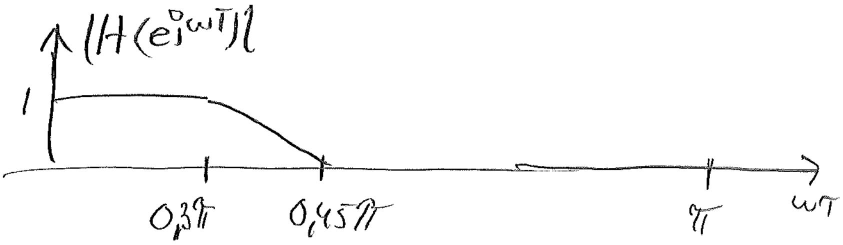
II: $\alpha_1 = \frac{2/R_3}{1/R_3 + 1/R_4 + 1/R_5} = 0.1457$

$\alpha_2 = 1.7084$

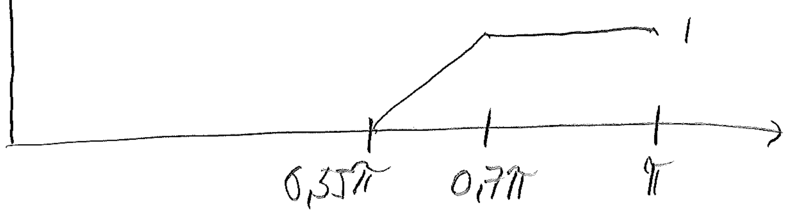
$\alpha_3 = \alpha_1 = 2 - \alpha_1 - \alpha_2 = 0.1457$

$R_3 = R_5$

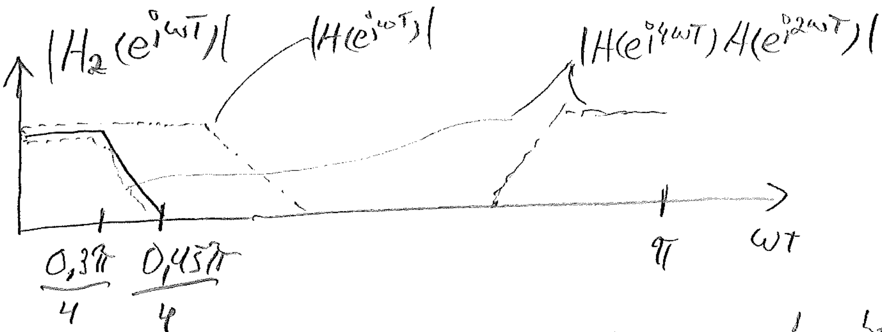
6)



a) $|H_1(e^{j\omega T})|$ $\left(z \to -z \Leftrightarrow \omega T \to \omega T \pm \pi, \text{ since } -1 = e^{\pm j\pi} \right)$



b)



c) $H_3(e^{j\omega T}) = 0$ for all ωT (No overlap between $H(z^3)$ and $H(-z)$ in the freq. domain)

7a)

Type	Mult	Add
I	$N/2+1$	N
II	$(N+1)/2$	N
III	$N/2$	$N-1$
IV	$(N+1)/2$	N

b) Type II and Type IV cascaded $\Rightarrow$

$$H(e^{j\omega T}) = H_{1R}(\omega T)\, e^{j\omega T N_1/2} \cdot H_2(\omega T) \cdot j \cdot e^{j\omega T N_1/2}$$

$$= \underbrace{H_{1R}(\omega T) \cdot H_{2R}(\omega T)}_{\text{Real}} \cdot j \cdot e^{j\omega T N/2}, \quad N = N_1 + N_2 = \text{even}$$

N_1 & N_2 odd

$\therefore$ Type III