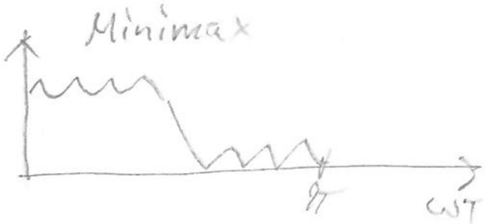
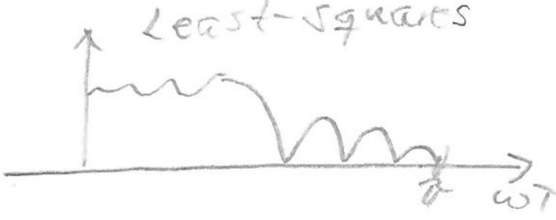
Digital Filters 170603 - Solutions TSE107

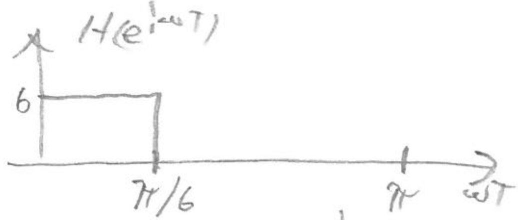
1a) To avoid delay-free loops

b)



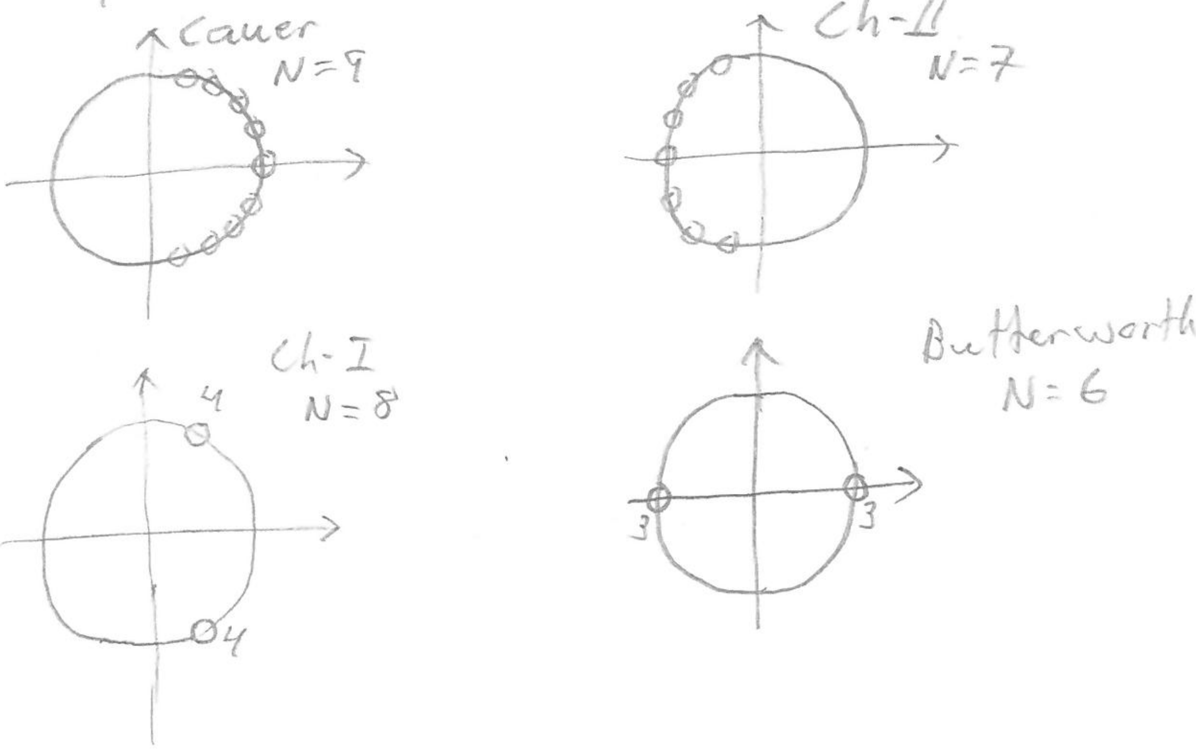


c) $\rightarrow \boxed{\uparrow 6} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 5} \rightarrow$



d) Type I $\left(j \cdot H_{R_1}(\omega T) \cdot j \cdot H_{R_2}(\omega T) = -H_{R_1}(\omega T) H_{R_2}(\omega T) \right)$

e)



2) Spec. of analog filter (Here $\frac{2}{T}=1$ used)

$\omega_{ac} = \tan\left(\frac{\omega_c T}{2}\right) = 3.0777$, $\omega_{as} = \tan\left(\frac{\omega_s T}{2}\right) = 1$

HP → LP spec.

$\Omega_c = \frac{\omega_I^2}{\omega_{ac}} = \left|\text{use } \omega_I^2 = \omega_{ac}\right| = 1$, $\Omega_s = \frac{\omega_{ac}}{\omega_{as}} = 3.0777$

Filter order $N=4$ (Ch-I, $\frac{\Omega_s}{\Omega_c} = 3{,}0777$)

Denormalized poles & zeros = denormalised since $\Omega_c = 1$

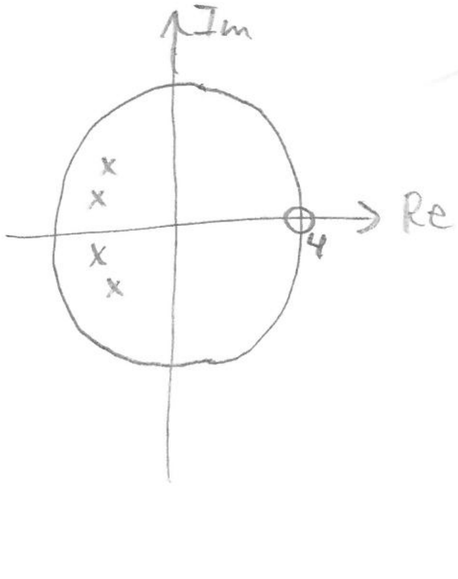
$$\left.\begin{array}{ll} \text{LP: zeros:} & 4 \text{ at } \infty \\ \text{poles:} & -0{,}1754 \pm j1.0163 \\ & -0{,}4233 \pm j0{,}4209 \end{array}\right\} \mathcal{S}\text{-domain}$$

LP → HP $\quad \mathcal{S} = \frac{\omega_I^2}{s} = \frac{\omega_{ac}}{s}$

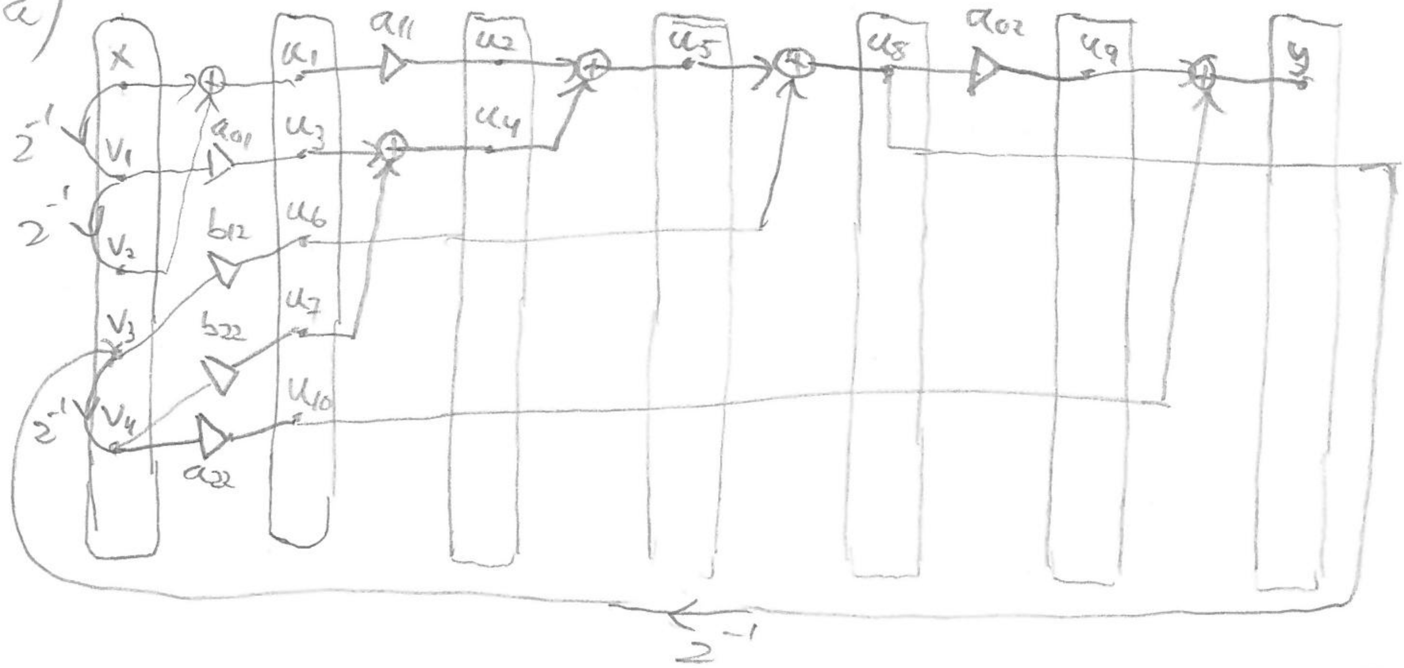
$$\left.\begin{array}{ll} \text{HP: zeros:} & 4 \text{ at } s=0 \\ \text{poles:} & -0{,}5074 \pm j2.9409 \\ & -3{,}6536 \pm j3.6349 \end{array}\right\} s\text{-domain}$$

$H_a(s) \to H(z)$, $s = \frac{z-1}{z+1}$, $z = \frac{1+s}{1-s}$ $\left(\frac{2}{T}=1\right)$

$$\left.\begin{array}{ll} \text{zeros:} & 4 \text{ at } z=1 \\ \text{poles:} & -0{,}7239 \pm j0{,}5386 \\ & -0{,}7331 \pm j0{,}2084 \end{array}\right\} z\text{-domain}$$



3a)

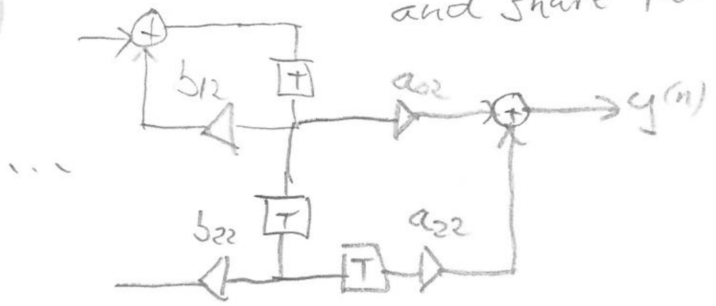


b) $T_{min} = \max\left\{ \frac{T_{mult}+T_{add}}{1}, \frac{T_{mult}+3T_{add}}{2} \right\} = T_{mult}+T_{add} = 1{,}25\,\mu s$

$T_{CP} = 2T_{mult} + 4T_{add} = 3\,\mu s$

Propagate one delay elem. from the output into the inputs of a_{02} and a_{22} and share two delay elem.

c)



4) $\omega_c T = 2\pi \cdot \frac{f_c}{f_{sample}} = 0{,}3\pi$, $\omega_s T = 2\pi \cdot \frac{f_s}{f_{sample}} = 0{,}75\pi$

$\omega_{ac} = \tan\left(\frac{\omega_c T}{2}\right) = 0{,}5095$, $\omega_{as} = \tan\left(\frac{\omega_s T}{2}\right) = 2{,}4142$

$$\left.\begin{array}{l} \text{Cauer}, \; \frac{\omega_{as}}{\omega_{ac}} = 4{,}7382 \\ A_{min} = 40, A_{max} = 0{,}1 \end{array}\right\} \Rightarrow \text{order } N = 3$$

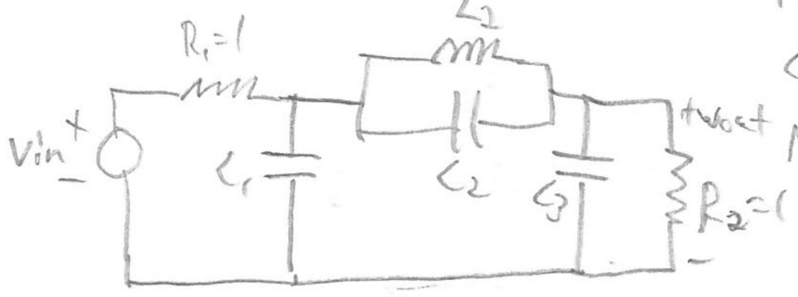
Table p. 27, $13 \leq \Theta \leq 16$

choose, e.g., $\Theta = 15$

Normalized: $C_1' = C_3' = 0{,}9944$

$C_2' = 0{,}04631$

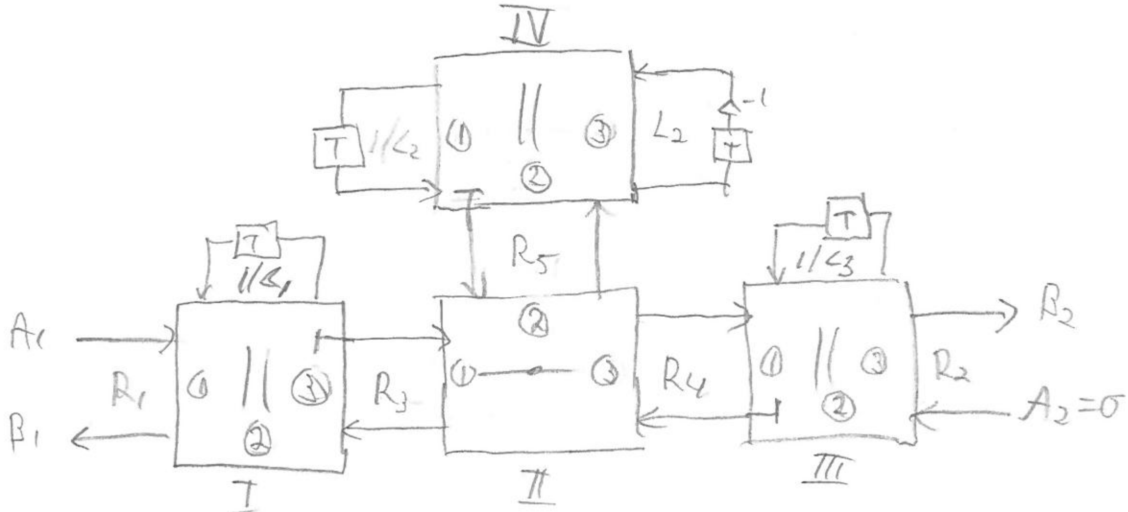
$L_2' = 1.0941$



Denormalize, divide by ω_{ac} (R=1)

$\Rightarrow C_1 = C_3 = 1{,}9516$, $C_2 = 0{,}0909$

$L_2 = 2{,}1473$



I: $G_3 = \frac{1}{R_1} + C_1$

$\alpha_1 = \frac{1/R_1}{1/R_1 + C_1} = 0{,}3388$

$\alpha_2 = 1 - \alpha_1 = 0{,}6612$

$\alpha_3 = 1$

III: symm. with I: $G_4 = G_3$

$\alpha_1 = 1$

$\alpha_2 = 0{,}6612$

$\alpha_3 = 0{,}3388$

IV: $G_5 = \frac{1}{L_2} + C_2 = 0{,}5566$

$\alpha_1 = \frac{2C_2}{C_2 + G_5 + 1/L_2} = 0{,}1633$

$\alpha_2 = 1$, $\alpha_3 = 1 - \alpha_1 = 0{,}8367$

II: $\alpha_1 = \frac{2R_3}{R_3 + R_5 + R_4} = 0{,}2739$ $\left(R = \frac{1}{G}\right)$

$\alpha_2 = 2 - 2\alpha_1 = 1{,}4523$

$\alpha_3 = \alpha_1$

5a) $\sigma_e^2 = \frac{Q^2}{12} \times 3 \times \sum_{n=0}^{\infty} g^2(n) = \left/ \begin{array}{l} G(z) = \frac{1}{1-0{,}625z^{-1}} \\ g(n) = 0{,}625^n \cdot u(n) \end{array} \right/$

$= \frac{Q^2}{12} \times 3 \times \frac{1}{1-0{,}625^2} = \left/ Q = 2^{-14} \right/ = 1.5283 \times 10^{-9}$

b) $x(n)$ scaled $\Rightarrow$ inputs to leftmost mult. scaled
only the output needs to be scaled – done by multiplying
the leftmost mult. by $c = \frac{1}{\sqrt{\sum_{n=0}^{\infty} h^2(n)}}$

$H(z) = \frac{0{,}575(1-z^{-2})}{1-0{,}625z^{-1}} \Leftrightarrow h(n) = 0{,}575 \cdot 0{,}625^n \cdot u(n) - 0{,}575 \cdot 0{,}625^{n-2} u(n-2) \Rightarrow$

$\Rightarrow h(n) = \begin{cases} 0{,}575, & n=0 \\ 0{,}575 \times 0{,}625, & n=1 \\ 0{,}575(1-0{,}625^{-2}) \cdot 0{,}625^n, & n \geq 2 \end{cases}$

$0{,}575 \times 0{,}625 = 0{,}359375$

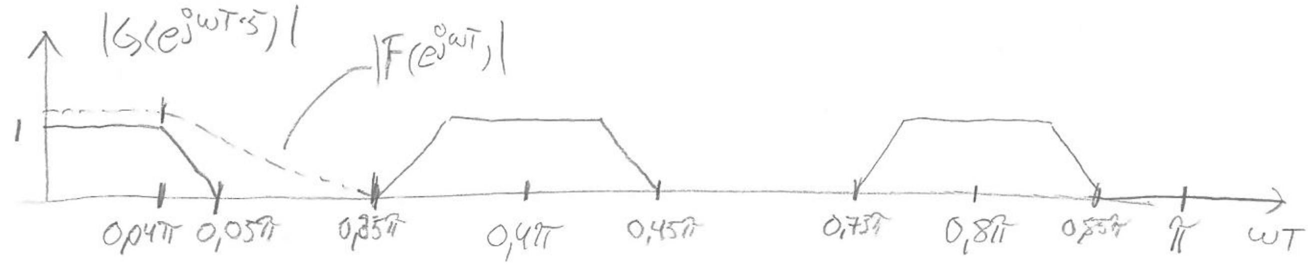
$0{,}575(1-0{,}625^{-2}) = -0{,}897 = A$

$\Rightarrow \sum_{n=0}^{\infty} h^2(n) = h^2(0) + h^2(1) + \sum_{n=2}^{\infty} A^2 (0{,}625^2)^n$

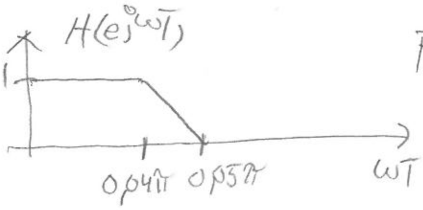
$= h^2(0) + h^2(1) + A^2 \cdot 0{,}625^4 \cdot \frac{1}{1-0{,}625^2} = 0{,}66125$

$\Rightarrow c = \frac{1}{\sqrt{0{,}66125}} \approx 1{,}2298$

6a)



b)



F: see above

c) $H: \delta_c^H, \delta_s^H$

$G \& F$ – stopband: $\delta_s^G = \delta_s^F = \delta_s^H$

passband: $\delta_c^F + \delta_c^G = \delta_c^H$ (approx, δ^2-terms ignored)

(For ex. $\delta_c^F = \delta_c^G = \frac{\delta_c^H}{2}$

d) Replace each delay elem. with M delay elem. in cascade

7a) $H_1(z) = \frac{-\alpha_0 z + 1}{z - \alpha_0} = \frac{-\alpha_0 + z^{-1}}{1 - \alpha_0 z^{-1}}$

$$H_2(z) = \frac{-\alpha_1 + A(z)}{1 - \alpha_1 A(z)}, \quad A(z) = z^{-1} \cdot \frac{-\alpha_2 + z^{-1}}{1 - \alpha_2 z^{-1}} = \frac{-\alpha_2 z^{-1} + z^{-2}}{1 - \alpha_2 z^{-1}}$$

$$= \frac{-\alpha_1 + \frac{-\alpha_2 \cdot z^{-1} + z^{-2})}{1 - \alpha_2 z^{-1}}}{1 - \alpha_1 \cdot \frac{-\alpha_2 \cdot z^{-1} + z^{-2}}{1 - \alpha_2 z^{-1}}} = \frac{-\alpha_1 + \alpha_1 \alpha_2 z^{-1} - \alpha_2 z^{-1} + z^{-2}}{1 - \alpha_2 z^{-1} + \alpha_1 \alpha_2 z^{-1} - \alpha_1 z^{-2}} =$$

$$= \frac{-\alpha_1 - \alpha_2 (1 - \alpha_1) z^{-1} + z^{-2}}{1 - \alpha_2 (1 - \alpha_1) z^{-1} - \alpha_1 z^{-2}} = \frac{-\alpha_1 z^2 - \alpha_2 (1 - \alpha_1) z + 1}{z^2 - \alpha_2 (1 - \alpha_1) z - \alpha_1}$$

b) $z - p_0 = z - \alpha_0 \Rightarrow \alpha_0 = p_0 = 0{,}575385$

$(z - p_1)(z - p_2) = z^2 - 2\,\mathrm{Re}\{p_1\} z + |p_1|^2 = z^2 - \alpha_2 (1 - \alpha_1) z - \alpha_1$

$\Rightarrow \alpha_1 = -|p_1|^2 \approx -0{,}6595$

$\alpha_2 = \frac{2\,\mathrm{Re}\{p_1\}}{1 + |p_1|^2} \approx 0{,}6715$