Digital Filters, 160601 - Solutions

1 a) $H(e^{j\omega T}) = H_R(\omega T)\, e^{j\Theta(\omega T)}$, $|H| = |H_R|$

real-valued zero-phase freq. resp.

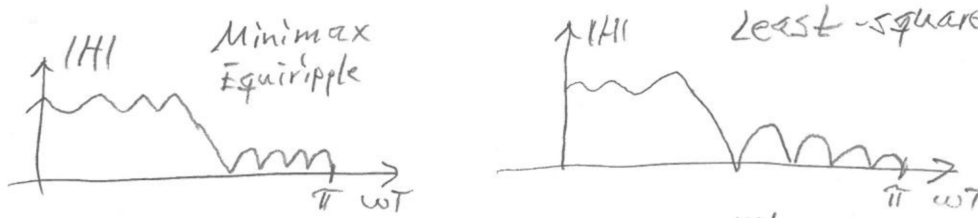
b) Ladder WDF - low sensitivity in passb. & stopb.

Lattice WDF - low sensit. in passb.
high — " — stopb.

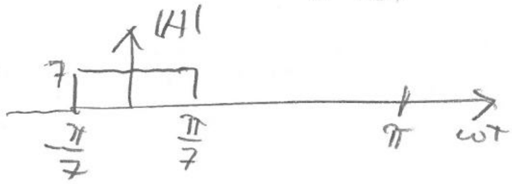
c) recursive - "feedback", nonrecursive - "feed forward"

IIR always recursive, FIR normally nonrecursive
can be realized recursively

d)



e) $\rightarrow \boxed{\uparrow 7} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 2} \rightarrow$



2) Spec. of analog HP: $\omega_{ac} = \frac{2}{T}\cdot\tan\left(\frac{\omega_c T}{2}\right) \approx \frac{2}{T}\cdot 3{,}078$

$\omega_{as} = \frac{2}{T}\cdot\tan\left(\frac{\omega_s T}{2}\right) \approx \frac{2}{T}\cdot 0{,}680$

Spec of analog LP $\quad \Omega_{ac} = \frac{\omega_I^2}{\omega_{ac}} = \left| \text{set } \omega_I^2 = \omega_{ac} \right| = 1$

$\Omega_{as} = \frac{\omega_{ac}}{\omega_{as}} = 4{,}5287$

Normalized poles and zeros (Table, $N=3$, $\rho=15\%$, $\theta=13$)

Poles: $-0{,}997065$ $\quad$ Zeros: $\pm j\,3{,}116621$

$-0{,}466446 \pm j\,1{,}211374$ $\quad$ ∞

Denormalized = Normalized, $\Omega_{ac} = 1$

$LP \to HP$, $s_{HP} = \frac{\omega_I^2}{s_{LP}} \Rightarrow$

poles: $(-3{,}0867)\cdot\frac{T}{2}$ $\quad$ zeros $\pm j\,0{,}6015$

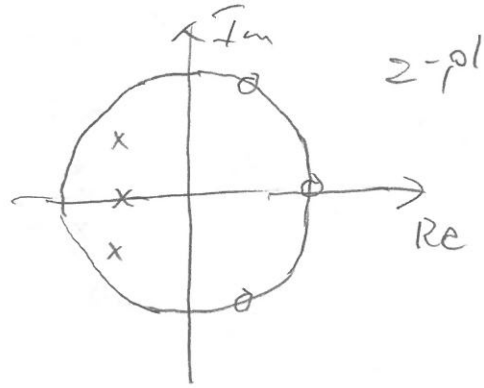
$(-0{,}8520 \pm j\,2{,}2126)\cdot\frac{T}{2}$ $\quad$ 0

Analog to digital, bilinear transf. $z = \frac{1+s\frac{T}{2}}{1-s\cdot\frac{T}{2}} \Rightarrow$

poles: $-0{,}510610$

$-0{,}555101 \pm j\,0{,}531530$

zeros: $0{,}468629 \pm j\,0{,}883395$



z-plane

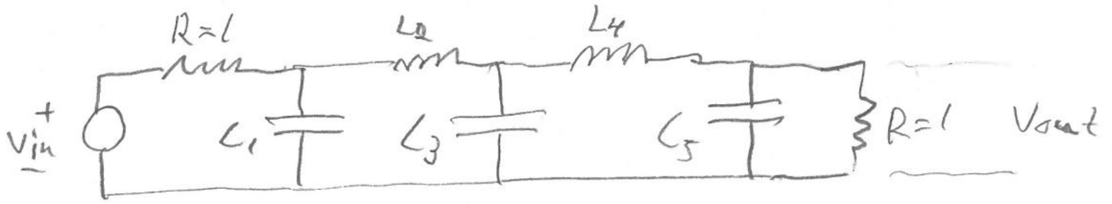
3) Spec of analog LP $\quad \omega_{ac} = \tan\left(\frac{\omega_c T}{2}\right) = 0,3249$

$\omega_{as} = \tan\left(\frac{\omega_s T}{2}\right) = 0,8541$

Filter order $N = 5$ $\quad$ (Ch. I, $\frac{\omega_{as}}{\omega_{ac}} = 2,63$)

π-net $\quad$ to keep it as simple as possible (R arbitrary)

For ex,



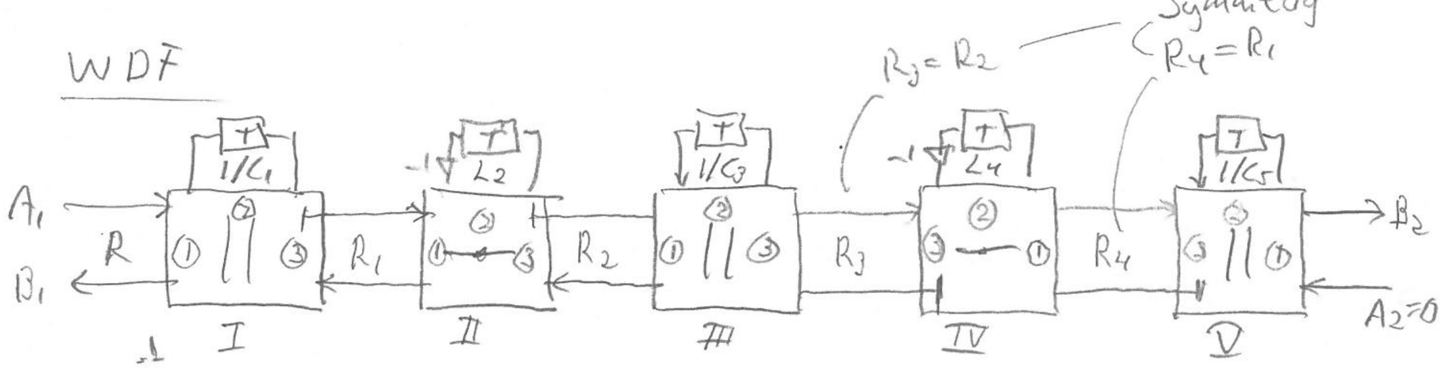
From Table: $\quad C_1' = C_5' = 1,1468$, $C_3' = 1,9750$

$L_2' = L_4' = 1,3712$

Denormalize $\quad$: $\quad C_1 = C_5 = 3,5294$, $C_3 = 6,0788$

mult by $\frac{1}{\omega_{ac}}$ $\quad L_2 = L_4 = 4,220$

WDF



I: $G_1 = \frac{1}{R} + C_1 = 4,5294$ $\quad \left(= \frac{1}{R_1}\right)$

$\alpha_1 = \frac{2 \cdot \frac{1}{R}}{\frac{1}{R} + C_1 + G_1} = 0,2208$

$\alpha_2 = 1 - \alpha_1 = 0,7792$

$\alpha_3 = 1$

II: $R_2 = R_1 + L_2 = 4,4410 = \frac{1}{G_2}$

$\alpha_1 = \frac{2R_1}{R_1 + L_2 + R_2} = 0,04971$

$\alpha_2 = 1 - \alpha_1 = 0,9503$

$\alpha_3 = 1$

III: $\alpha_1 = \frac{2G_2}{G_2 + C_3 + G_3} = 0,06898$ $\quad$ ($G_3 = G_2$)

$\alpha_2 = 2 - \alpha_1 - \alpha_3 = 2 - 2\alpha_1 = 1,8620$

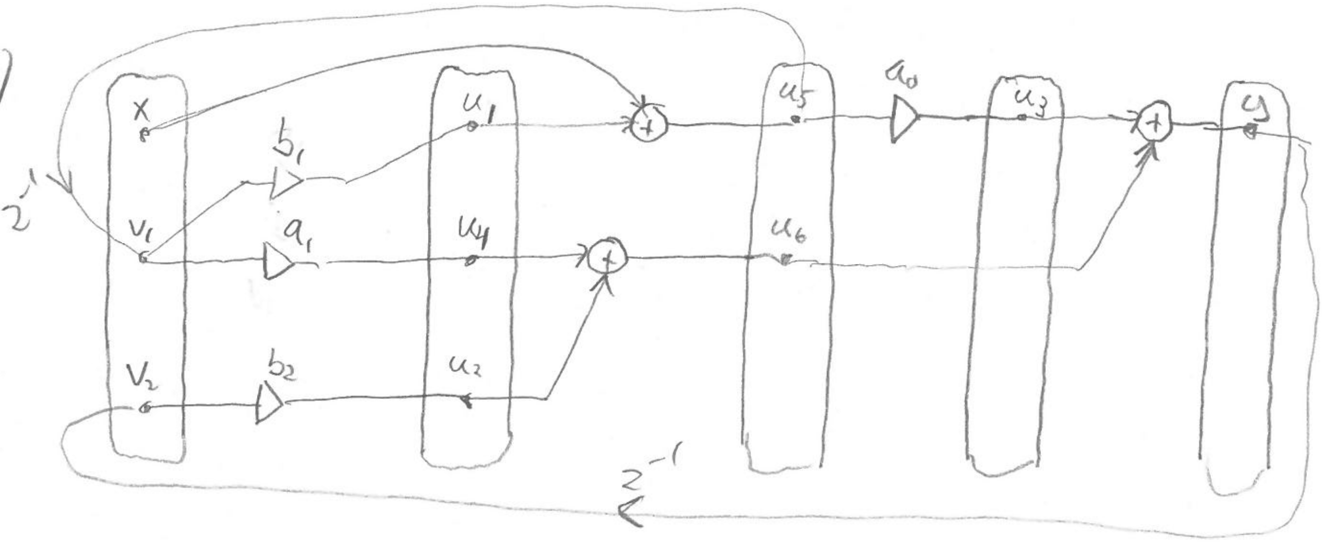
$\alpha_3 = \alpha_1$ (symmetry) $= 0,06898$

IV same as II

V same as I

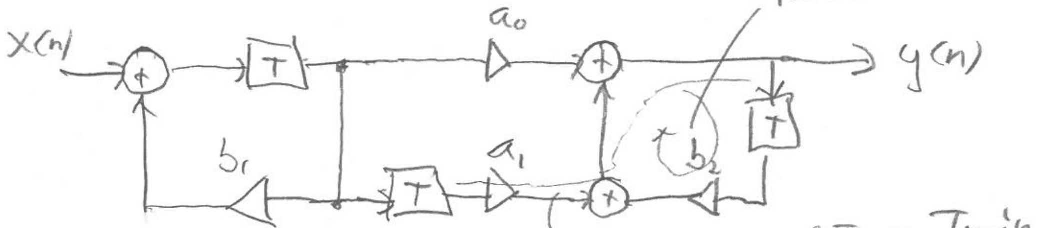
due to symmetry

4a)



b) Pipelined structure

$T_{min} = T_m + 2T_a$



$T_{cp} = T_m + 2T_a = T_{min}$

Propagete one delay elem, into the structure and

5a) Two roundings $\Rightarrow P_e = \underbrace{\frac{Q^2}{12}}_{\text{variance}} \cdot 2 \cdot \sum_{n=0}^{\infty} g^2(n)$, $g(n) = (-b)^n u(n)$

$G(z) = \dfrac{1}{1 + bz^{-1}}$

$$\Rightarrow P_e = \frac{Q^2}{12}\, 2 \cdot \sum_{n=0}^{\infty} (b^2)^n = \frac{Q^2}{12} \cdot \frac{2}{1-b^2} =$$

$$= \frac{2^{-20}}{6} \cdot \frac{1}{1-b^2} = 1.589 \cdot 10^{-7} \cdot \frac{1}{1-b^2}$$

b) Branch 2 allpass ($h_{ap}(n)$) $\Rightarrow$ Inputs to both mult scaled

Output: $\sum_{n=0}^{\infty} h^2(n) = (1 + h_{ap}(0))^2 + \underbrace{\sum_{n=1}^{\infty} h_{ap}^2(n)}_{= 1 - h_{ap}^2(0)} =$

since $\sum_{n=0}^{\infty} h_{ap}^2(n) = 1$

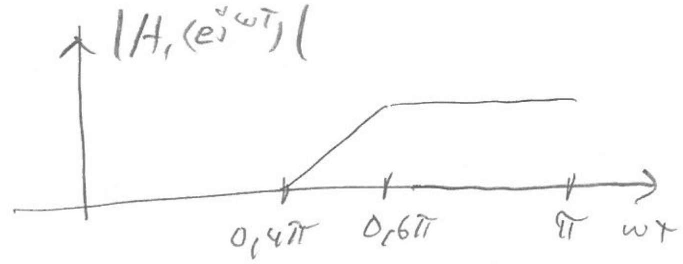
/ $h_{ap}(0) = b$ /

$$= (1+b)^2 + 1 - b^2$$

$$= 1 + 2b + b^2 + 1 - b^2 = 2(1-b)$$

$x(n) \xrightarrow{c} \dots \qquad c = \dfrac{1}{\sqrt{2(1-b)}}$

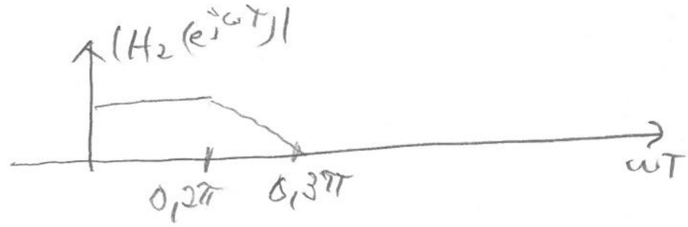
6a)



Filter order

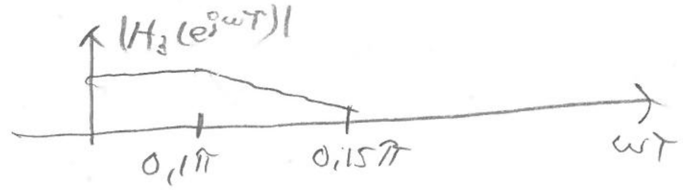
$N_1 = N$

b)



$N_2 = 3N$

c)



$N_3 = 7N$

7a) Type I : $\frac{N}{2}+1$ mult

II : $\frac{N+1}{2}$

III : $\frac{N}{2}$

IV : $\frac{N+1}{2}$

b) Two Type IV in cascade ⇒ Type I

$$\left(j \cdot H_{R1}(\omega T) \cdot e^{-j\omega T \cdot N_1/2}\right) \cdot \left(j\, H_{R2}(\omega T)\, e^{-j\omega T N_2/2}\right)$$

$$= \underbrace{- H_{R_1}(\omega T)\, H_{R_2}(\omega T)}_{H_R(\omega T)}\, e^{-j\omega T (N_1+N_2)/2}$$

And odd+odd = even order

c) Type III in cascade with Type I ⇒ Type III

$$(j \cdot \ldots) \cdot (1 \cdot \ldots)$$

$j \cdot 1 = j$ instead of $j \cdot j = -1$ as in b)

And even + even = even order