LINKÖPING UNIVERSITY

Exam in TSEI07 Digital Filters

Exam code:	TEN1
Date:	2019-05-29 **Time:** 14–18
Place:	TER3
Examiner:	Håkan Johansson
Department:	ISY
Allowed aids:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables
Number of tasks:	7
Grading:	Maximum 70 points, where 30, 42, and 56 points are required for the grades 3, 4, and 5, respectively. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE06/
Result:	Available by 2019-06-13

1 **a.** How are the group delay and phase delay related to the phase response of a filter? (2 p)

b. What does forced-response stability mean? (2 p)

c. What is the difference between recursive and nonrecursive structures? How are they related to FIR and IIR filters? (2 p)

d. Sketch typical magnitude responses for highpass filters designed in the minimax sense and least-squares sense, respectively. (2 p)

e. Indicate in the z-plane typical zero locations for the following digital filters: 5th-order lowpass Cauer filter, 9th-order highpass Chebyshev-II filter, 10th-order bandpass Butterworth filter, 8th-order bandstop Chebyshev-I filter. (2 p)

2 Synthesize a minimum-order Cauer filter that meets the specification below. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. (10 p)

$\omega_c T = 0.13\pi$, $\omega_s T = 0.5\pi$, $A_{\max} = 0.09883$ dB ($\rho = 15\%$), $A_{\min} = 40$ dB.

3 Realize a minimum-order Chebyshev-I T-type ladder wave digital filter that meets the specification below. Assume source and load resistor values $R_1 = R_2 = 1$. Use Richards variable $\Psi = \frac{z-1}{z+1}$ and directly interconnected adaptors with reflection-free ports. Draw the block diagram and compute the adaptor coefficients. (10 p)

$\omega_c T = 0.8\pi$, $\omega_s T = 0.55\pi$, $A_{\max} = 0.5$ dB, $A_{\min} = 50$ dB.

4 A filter structure is given according to Figure 1. It is composed of two first-order allpass sections in cascade, where each section is realized using a symmetric two-port adaptor.

a. Draw the signal-flow graph in precedence form. (6 p)

b. Determine the critical path (T_{CP}) and minimal sampling period (T_{min}), expressed in terms of T_{add} and T_{mult}. (2 p)

c. Use pipelining so that the new T_{CP} equals T_{min}. (2 p)

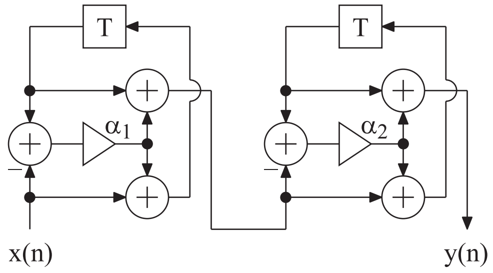

Figure 1:

5 **a.** Compute the variance of the roundoff noise at the output of the Nth-order causal linear-phase FIR filter structure seen in Figure 2. The value of N and the non-zero impulse response values $h(n)$ of the filter are given at the bottom of this page. Assume that quantization (rounding) is carried out after each multiplication, and that the quantizations can be modeled as uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-12}$. Utilize that, when white noise propagates through an LTI system with the impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain of an LTI system with the impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$\text{Noise Gain} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b. Scale the filter using safe scaling (see below) which ensures that the critical nodes will be within the same number range as the input signal. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. (5 p)

Safe scaling utilizes that the absolute value of a node $v(n)$ is bounded as $|v(n)| \leq \max\{|x(n)|\} \times \sum_n |f(n)|$, where $f(n)$ is the impulse response from the input signal $x(n)$ to the node $v(n)$.

$h(n) = n + 1, n = 0, 1, \ldots, N/2$
$h(n) = h(N - n), n = N/2 + 1, N/2 + 2 \ldots, N$
$N = 20$

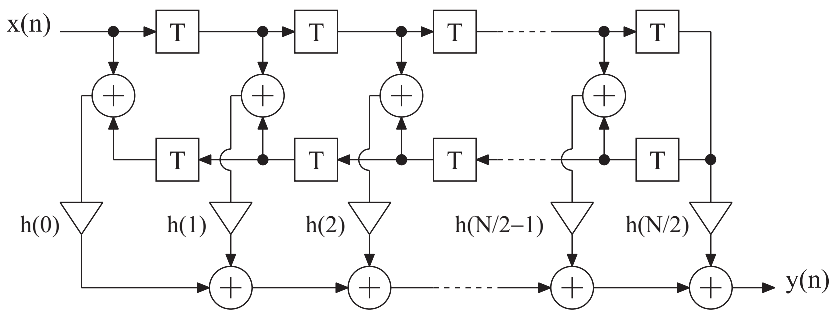

Figure 2:

6 A system for interpolation is shown in Fig. 3(b) where the input signal $x(n)$ has a real-valued spectrum according to Fig. 3(a).

a. Determine the overall interpolation factor. (2 p)

b. Determine the passband and stopband edges of the two filters $H_1(z)$ and $H_2(z)$ so that the spectral images that appear in the upsampling stages are completely eliminated and the desired spectrum is preserved (for simplicity it is assumed that the filters' passband and stopband ripples are zero.). The transition bands should be as wide as possible. Also determine the gain constants of the filters. (4 p)

c. Sketch the spectrum for the five signals $y_k(n), k = 1, 2, 3, 4, 5$. (4 p)

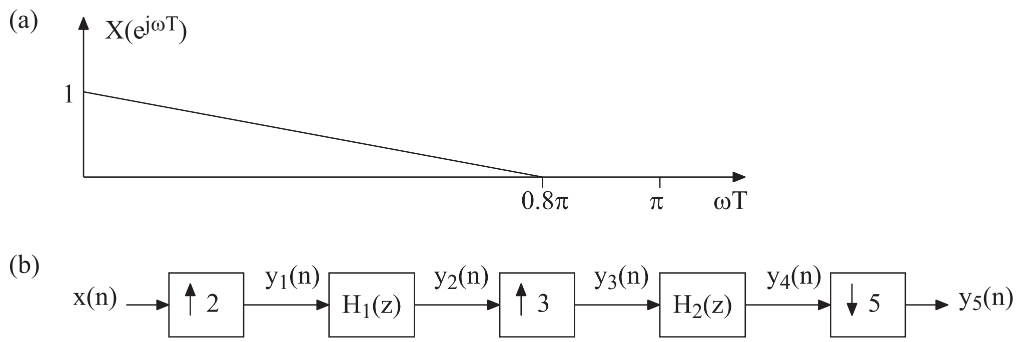

Figure 3:

7 There are four types of causal Nth-order linear-phase FIR filters as follows. Type I: symmetric impulse response and even N; Type II: symmetric impulse response and odd N; Type III: antisymmetric impulse response and even N; Type IV: antisymmetric impulse response and odd N.

a. For each of the four types, determine how many multipliers and delay elements (expressed as a function of N) that are needed in both a direct-form structure and a linear-phase direct-form structure. In the latter of these two structures, the impulse response symmetries/antisymmetries are utilized. (7 p)

b. What type of overall filter (Type I, II, III, or IV) is obtained if a Type III filter is cascaded with a Type IV filter? (3 p)