LINKÖPING UNIVERSITY

Exam in TSEI07 Digital Filters

Exam code:	TEN1
Date:	2018-05-30 **Time:** 14–18
Place:	TER2, TER3
Examiner:	Håkan Johansson
Department:	ISY
Allowed aids:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables
Number of tasks:	7
Grading:	Maximum 70 points, where 30, 42, and 56 points are required for the grades 3, 4, and 5, respectively. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE06/
Result:	Available by 2018-06-13

1 **a.** Explain what high sensitivity of a filter structure means and implies. (2 p)

b. Which are the critical nodes that need to be scaled in digital filters when two's-complement arithmetic is used? What is the difference between safe scaling and L_p-norm scaling? (2 p)

c. Why must some ports be reflection-free when realizing wave digital filters using directly interconnected adaptors? (2 p)

d. An FIR filter transfer function is given as: $H(z) = z^{-1} + 2z^{-3} + z^{-5}$. Does the filter have a linear phase response? (2 p)

e. Indicate in the z-plane typical zero locations for the following digital filters: 7th-order highpass Cauer filter, 5th-order lowpass Chebyshev-II filter, 8th-order bandpass Chebyshev-I filter, 10th-order bandstop Butterworth filter.

(2 p)

2 Synthesize a digital Chebyshev-I filter that meets the attenuation specification in Figure 1. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. (10 p)

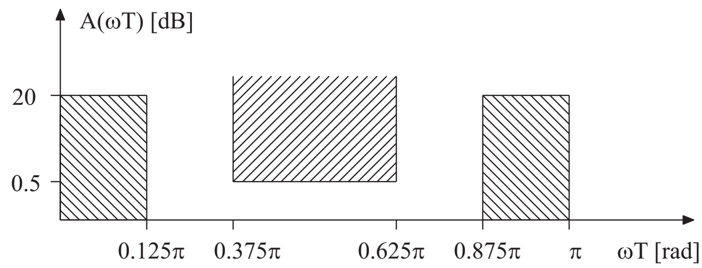

Figure 1:

3 A filter structure is given according to Figure 2.

a. Draw the signal-flow graph in precedence form. (6 p)

b. Determine the minimal sampling period, T_{min}, in terms of T_{mult} and T_{add}. (1 p)

c. Use pipelining so that the critical path, T_{CP}, of the pipelined structure contains one multiplier and two adders. The pipelined structure should not have more than four delay elements in total. (3 p)

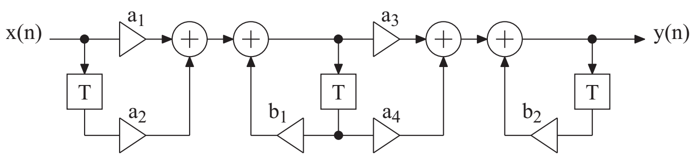

Figure 2:

4 **a.** Compute the variance of the roundoff noise at the output of the filter seen in Figure 3. Assume that quantization (rounding) is carried out after each multiplication, and that the quantizations can be modelled as uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-12}$. Utilize that, when white noise propagates through an LTI-system with the impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain of an LTI system with the impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$\text{Noise Gain} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b. Scale the filter using the L_2-norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. (5 p)

L_2-norm of an LTI system with the impulse response $f(n)$ and frequency response $F(e^{j\omega T})$:

$$||F(e^{j\omega T})||_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

5 Realize a third-order Cauer wave digital filter satisfying the highpass filter requirements below. Start from a lowpass filter that is realized with a T-net with load and generator resistances of 1 Ω. Use Richards variable $\Psi = (z-1)/(z+1)$ and directly interconnected adaptors with reflection-free ports. Draw the wave-flow graph (block diagram) and compute the adaptor coefficients. (10 p)

$\omega_c T = 0.8\pi$, $\omega_s T = 0.3\pi$, $A_{\max} = 0.1$ dB ($\rho = 15\%$), $A_{\min} = 50$ dB.

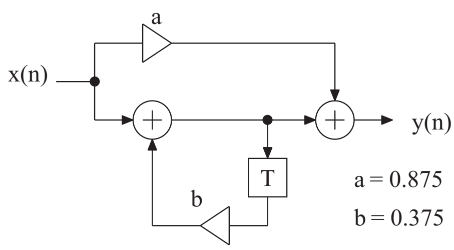

Figure 3:

6 Assume that the filter $H(z)$ is a lowpass filter with passband and stopband edges at 0.3π and 0.45π, respectively. For simplicity, also assume that the gain is unity in the passband and zero in the stopband. Sketch the magnitude responses of the three filters $H_1(z)$, $H_2(z)$, and $H_3(z)$, as given below. Also, for each filter, determine the filter order (expressed as a function of N), assuming that the filter order of $H(z)$ is N.

a. $H_1(z) = H(-z)$ (2 p)

b. $H_2(z) = H(z^4)H(z^2)H(z)$ (4 p)

c. $H_3(z) = H(z^3)H(-z)$ (4 p)

7 There are four types of causal Nth-order linear-phase FIR filters as follows. Type I: symmetric impulse response and even N; Type II: symmetric impulse response and odd N; Type III: antisymmetric impulse response and even N; Type IV: antisymmetric impulse response and odd N.

a. For each of the four types, determine how many multipliers and adders (expressed as a function of N) that are needed in a direct-form linear-phase FIR structure, utilizing the impulse response symmetries and antisymmetries. (7 p)

b. What type of overall filter (Type I, II, III, or IV) is obtained if a Type II filter is cascaded with a Type IV filter? (3 p)