LECTURE 1, Digital Filters

Introduction

On the White Board

Basic Concepts, Definitions, and Notations

Continuous-time signals and systems

Discrete-time signals and systems

LTI-systems

Difference equation, Impulse response

Frequency response, Transfer function

Realization/structure/algorithm

Digital Filters

Frequency selective LTI digital filters

Digital filtering of analog signals (sampling, filtering, reconstruction)

Applications

Frequency division multiplexing/demultiplexing

Noise suppression

Subband coding

Design process

Synthesis, realization, implementation

Specification

Magnitude response, attenuation function

Phase response, group delay, phase delay

LECTURE 2, Digital Filters

DSP Algorithms

Difference equation

Realization/structure/algorithm

Signal flow graph (SFG) in precedence form

Computation graph - SFG plus execution times

Iterative processing

SFGs in Precedence Form

Four-step systematic procedure

Difference Equations in Computable Order

Obtained from SFG in precedence form

Computer Program

Obtained from difference equations in computable order

Maximal sample frequency (maximal data rate)

Recursive algorithms - upper bound

Nonrecursive algorithms - no upper bound

Critical path - longest path in computation graph

Pipelining

On the White Board

- Iterative processing
- SFGs in precedence form
- Difference equations in computable order
- Computer program
- Maximal sample frequency
  - Recursive algorithms - upper bound
  - Critical path
  - Pipelining

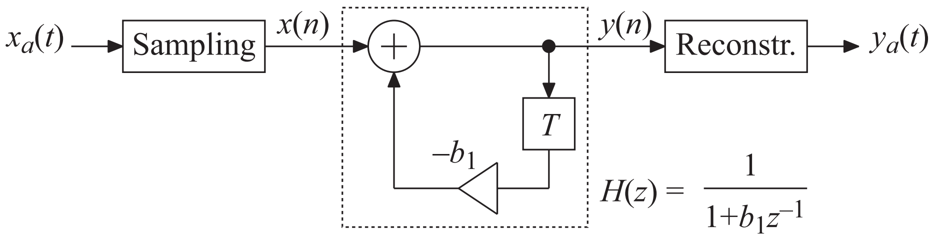
LECTURE 3, Digital Filters

Finite Wordlength Effects

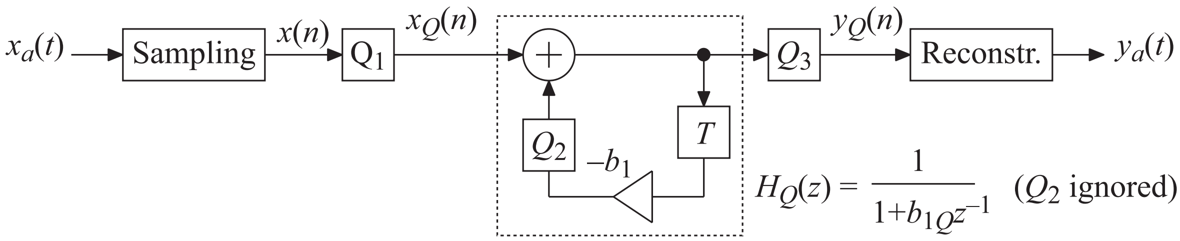
Ideal linear systems - real numbers

Actual digital systems - quantized numbers <=>
Weakly nonlinear systems

(a) Ideal linear system



(b) Actual nonlinear system



Fixed-point arithmetic - cheaper implementation

Two's-complement representation common

Quantization errors

- Overflow errors (large errors) - number range exceeded
- Rounding/truncation errors (small errors) - least significant bits

Errors in Digital Systems due to Quantization

1) Errors in the A/D and D/A conversions

2) Overflow errors - signal scaling required

3) Round-off errors - SNR at output common measure

4) Coefficient errors - change of filter function $H(z)$

Design Choices

Errors due to 2), 3), and 4) affected by the filter structure

Important to choose structures with good numerical properties

Nonrecursive structures - FIR

Wave digital filters - IIR

(Certain state-space structures, special recursive FIR filters with cancelling zeros and poles at $z = +/- 1$, not in focus in this course)

On the White Board

Fixed-point two's complement arithmetic

Overflow errors

- Parasitic overflow (large-scale) oscillations
- Forced-response stable filters

Scaling of signal levels - to avoid (reduce the risk of) overflow

- Safe scaling, *Lp*-norm scaling

Round-off errors

- Parasitic granularity (small-scale) oscillations - limit cycles
- Round-off noise - linear model of quantizer, white noise assumption

Coefficient errors

- Low-sensitivity structures

Conclusion

Use digital filter structures that

1) are stable under finite arithmetic (nonlinear) conditions

2) have low round-off noise - short data wordlength

3) have low sensitivity - short coefficient wordlength

=> robust and low implementation cost

Nonrecursive FIR

Wave digital filters

LECTURE 4, Digital Filters

FIR Filters

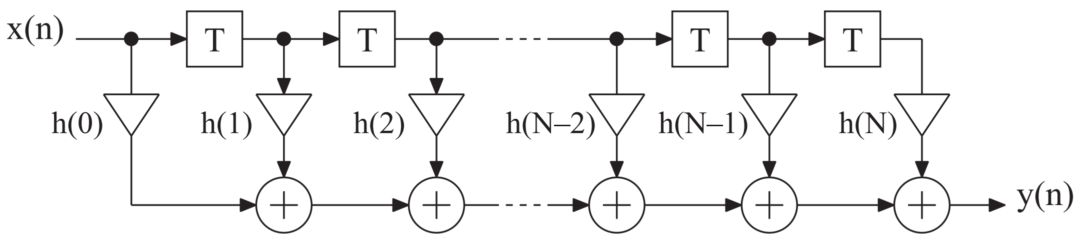
Transfer function: $H(z) = \sum_{n=0}^{N} h(n) z^{-n}$, Nth-order causal filter

Frequency response: $H(e^{j\omega T}) = \sum_{n=0}^{N} h(n) e^{-j\omega T n}$

+ Linear phase possible - (anti)symmetric impulse response

+ Can be realized with nonrecursive structures

Direct form: $y(n) = \sum_{k=0}^{N} x(n-k) h(k)$



- High order and complexity, and long delay, for stringent specifications

On the White Board

- Linear-Phase FIR filters - Types I, II, III, IV
- Zero locations
- Frequency response
  - Delay term and zero-phase frequency response
- Synthesis (process of specification -> filter order and coefficients)
  - Specification
  - Synthesis methods (windowing, optimization)
  - Approximation problem
  - Minimax design
  - Least-squares design

LECTURE 5, Digital Filters

Synthesis of Analog Filters

Specification -> $H_a(s) = \frac{\sum_{k=0}^{N} a_k s^k}{\sum_{k=0}^{N} b_k s^k} = G\frac{\prod_{k=1}^{N}(s-z_k)}{\prod_{k=1}^{N}(s-p_k)}$

z_k zeros, p_k poles, G gain constant (numerically preferred repres.)

Used in the Design of Digital IIR Filters

1) Spec. -> $H_a(s)$ -> (transformation) $H(z)$ -> realization

2) Spec. -> $H_a(s)$ and realization -> $H(z)$ and realization - WDF

Synthesis Methods for Analog Filters

Analytical - closed form solutions - standard approximations

Requirements only on the magnitude response $|H_a(j\omega)|$

Optimal lowpass and highpass filters. Suboptimal bandpass, bandstop, and multiband filters

Optimization - iterative methods

Optimal filters for all types (potentially, local minima possible)

More flexible - can handle requirements on the magnitude response, phase response, group delay response, etc.

On the White Board

Synthesis of lowpass filters

Standard approximations

Synthesis procedure

Synthesis of highpass, bandpass, and bandstop filters

Synthesis procedure

Frequency transformations

Focus in the Course

Basic principles and properties of the filter approximations
(not the whole underlying mathematical theory)

Parts from the analog filter design world needed for the synthesis of digital IIR filters

LECTURE 6, Digital Filters

Synthesis of Digital IIR Filters

Specification -> $$H(z) = \frac{\sum_{k=0}^{N} a_k z^{-k}}{\sum_{k=0}^{N} b_k z^{-k}} = G\frac{\prod_{k=1}^{N} (z - z_k)}{\prod_{k=1}^{N} (z - p_k)}$$

z_k zeros, p_k poles, G gain constant (numerically preferred repres.)

Synthesis Methods

Analytical - closed form solutions - standard approximations

Requirements only on the magnitude response $|H(e^{j\omega T})|$

Optimal lowpass and highpass filters. Suboptimal bandpass, bandstop, and multiband filters

Common to transform analog filter into digital IIR filter

Bilinear transformation - for requirements on $|H(e^{j\omega T})|$ only

Optimization - iterative methods

Optimal filters for all types (potentially, local minimum possible)

More flexible - can handle requirements on the magnitude response, phase response, group delay response, etc.

Nonlinear optimization - local minimum is a problem

a) Optimize $H_a(s)$ -> (transformation) -> $H(z)$

b) Optimize $H(z)$ directly

On the White Board

Bilinear transformation

Synthesis procedure

Deterioration of the phase response

LECTURE 7, Digital Filters

Multirate Systems

Systems that work with several different sampling frequencies

Two Approaches

Reconstruction to analog signal followed by resampling

Requires analog signal processing (ADC, filter, DAC)

Digital interpolation and decimation - preferred

Applications

Interconnection of systems with different sampling frequencies

Oversampling techniques - relax requirements on the analog parts

Reduction of computational complexity

Frequency Masking Techniques

Single-rate structures

Make use of interpolated impulse responses <=> periodic subfilters
=> sparse subfilters => few non-zero impulse response values
=> computationally efficient

On the White Board

- Interpolation (sampling rate increase)
- Decimation (sampling rate decrease)
- Transmultiplexers
  - Transmit information in different frequency bands
- Polyphase decomposition
  - Efficient interpolation and decimation structures
- Frequency masking techniques

LECTURE 8, Digital Filters

Wave Digital Filters (WDFs)

Obtained from a transmission line filter $H_a(s)$ via $z = e^{sT}$ or, equivalently, from a lumped reference filter $H_{ar}(\Psi)$ via $\Psi = \frac{z-1}{z+1}$

Inherit the good properties of the analog reference filters

Spec. -> $H_{ar}(\Psi)$ and realization -> $H(z)$ and realization - WDF

Doubly Resistively Terminated Reference Filters

Inductors, capacitors

Unit elements

Filter Structures

Ladder structures => ladder WDFs, low-sensitivity

Lattice structures => lattice WDFs, low sensitivity in the passband
high sensitivity in the stopband

Richards' structures - restricted to certain filter types

Basic principles and properties of the filter structures
(not the whole underlying mathematical theory)

Parts from the analog filter structure world needed for the realization of WDFs

On the White Board

Wave Digital Filters

Relations between the WDF, transmission-line filter, and Ψ-domain reference filter

Doubly resistively terminated filters

Filter structures

Ladder, Lattice, Richard's

Ladder structures

LP T-net and π-net with and without finite zeros

Computation of element values

HP, BP, and BS - frequency transformations

LECTURE 9, Digital Filters

Wave Digital Filters (WDFs)- Design Procedure

Three-step procedure

Specification mapping from the WDF to the Ψ-domain reference filter

Realization of the reference filter

Transformation of the reference filter into the WDF - voltage waves

Transformation of reference filter into WDF

1) Transformation of elements and sources

2) Interconnection of elements and sources - requires adaptors

Parallel connections - parallel adaptors

Series connections - series adaptors

On the White Board

Wave digital filters

Design procedure

Transformation of reference filter into WDF

Parallel and series adaptors

LECTURE 10, Digital Filters

Wave Digital Filters (WDFs) - Filter Realizations

Delay-Free Loops Must be Avoided

Three methods

1) Cascaded unit elements

2) Separating unit elements between inductors and capacitors

3) Reflection-free ports

WDF Structures

Richards WDFs - cascaded unit elements

Ladder WDFs - reflection-free ports or separating unit elements

Lattice WDFs - realize reactance
Foster, Cauer, Richards, Circulators

On the White Board

Richards WDFs

Ladder WDFs

Lattice WDFs