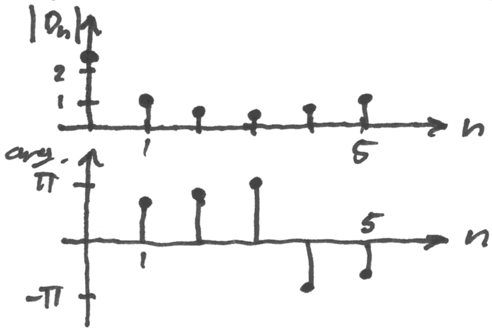
TSKS21 Facit till lektion 6

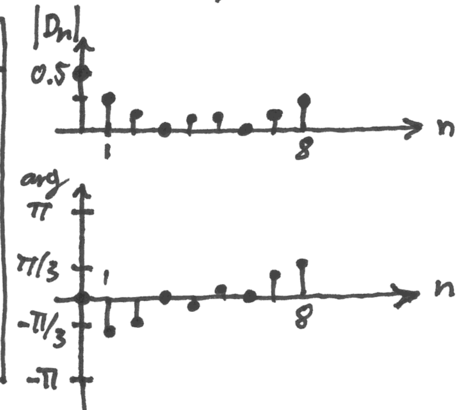
11.1 i) $D_n = \cos\left(\frac{\pi}{3}n\right) + \cos\left(\frac{2\pi}{3}n\right) + \frac{(-1)^n}{2} + j\left(\frac{2}{3}\sin\left(\frac{\pi}{3}n\right) + \frac{1}{3}\sin\left(\frac{2\pi}{3}n\right)\right)$

n	D_n	$\lvert D_n \rvert$	$\arg\{D_n\}$
0	$5/2$	$5/2$	0
1	$-1/2 + j\sqrt{3}/2$	1	$2\pi/3$
2	$-1/2 + j\sqrt{3}/6$	$1/\sqrt{3}$	$5\pi/6$
3	$-1/2$	$1/2$	π
4	$-1/2 - j\sqrt{3}/6$	$1/\sqrt{3}$	$-5\pi/6$
5	$-1/2 - j\sqrt{3}/2$	1	$-2\pi/3$

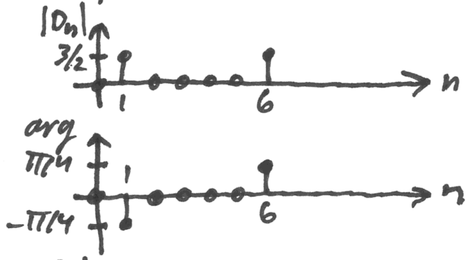


ii) $D_n = \frac{1}{18} e^{-j\frac{2\pi}{9}n}\left(2 + e^{-j\frac{2\pi}{3}n}\right)\left(1 + 2\cos\left(\frac{2\pi}{9}n\right)\right)$

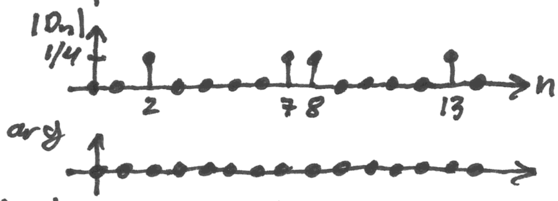
n	D_n	$\lvert D_n \rvert$	$\arg\{D_n\}$
0	0.500	0.500	0
1	$0.083 - j0.229$	0.244	$-7\pi/18$
2	$0.083 - j0.099$	0.130	$-5\pi/18$
3	0	0	0
4	$0.083 - j0.015$	0.085	$-\pi/18$
5	$0.083 + j0.015$	0.085	$\pi/18$
6	0	0	0
7	$0.083 + j0.099$	0.130	$5\pi/18$
8	$0.083 + j0.229$	0.244	$7\pi/18$



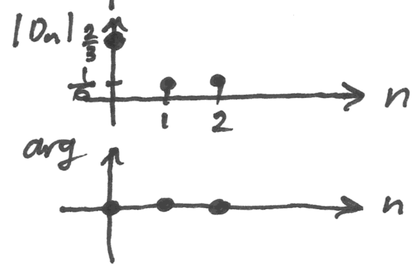
iii) $D_n = \begin{cases} \frac{3}{2}e^{j\pi/4}, & n = -1 \mod 7 \\ \frac{3}{2}e^{-j\pi/4}, & n = 1 \mod 7 \\ 0, & \text{f.ö.} \end{cases}$



vi) $D_n = \begin{cases} 1/4, & n = \pm 2 \text{ \& } n = \pm 7 \mod 15 \\ 0, & \text{f.ö.} \end{cases}$



vii) $D_n = \begin{cases} 2/3, & n = 0 \mod 3 \\ 1/6, & n = \pm 1 \mod 3 \end{cases}$



11.2 i) $x[k] = \begin{cases} 4.500, & k = 0 \mod 9 \\ 0.750 + j\,2.061, & 1 \\ 0.750 + j\,0.894, & 2 \\ 0, & 3 \\ 0.750 + j\,0.132, & 4 \\ 0.750 - j\,0.132, & 5 \\ 0, & 6 \\ 0.750 - j\,0.894, & 7 \\ 0.750 - j\,2.061, & k = 8 \mod 9 \end{cases}$ (jfr. 11.1 ii)

ii) $x[k] = 1 + 2\cos\left(\frac{2\pi}{7}k\right) - \sin\left(\frac{2\pi}{7}k\right)$

$= 1 + \sqrt{5}\cos\left(\frac{2\pi}{7}k + \arctan\left(\frac{1}{2}\right)\right)$

11.5 i) $\mathcal{F}\{(-1)^k x_1[k]\} = X_1(\Omega - \pi)$

ii) $\mathcal{F}\{(k-5)^2 x_2[k-4]\} = \left(j^2 \frac{d^2}{d\Omega^2} - j10\frac{d}{d\Omega} + 25\right)\left(X_2(\Omega)e^{-j4\Omega}\right)$

Tips: Studera $\frac{d}{d\Omega}\mathcal{F}\{x[k]\}$.

v) $\mathcal{F}\{x_1[5-k]x_2[7-k]\} = \mathcal{F}\{x_1[k+5]x_2[k+7]\}^*$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1^*(\theta)e^{-j5\theta} \cdot X_2^*(\Omega-\theta)e^{-j7(\Omega-\theta)}\, d\theta$

Allmänt gäller för reellvärt $x[k]$: $\mathcal{F}\{x[-k]\} = X^*(\Omega)$

Inses genom identifiering i def. av fouriertransformen.

11.6 i) $x[k] = 4(2^k - 3^k)u[-k]$

11.11 i) $h[k] = \left(\frac{9}{8}\cdot 0.3^k - \frac{25}{4}\cdot 0.5^k + \frac{49}{8}\cdot 0.7^k\right)u[k]$

ii) $-0.105y[k-3] + 0.71y[k-2] - 1.5y[k-1] + y[k] = x[k]$

11.12 i) $H(\Omega) = \dfrac{1 - e^{-j2\Omega}}{1 + e^{-j\Omega} + \frac{1}{4}e^{-j2\Omega}} = \dfrac{(1+e^{-j\Omega})(1-e^{-j\Omega})}{(1+\frac{1}{2}e^{-j\Omega})^2}$

ii) $h[k] = -4\delta[k] + 4\left(\frac{1}{2}\right)^{k-1}\cdot k \cdot u[k-1] + 5\left(\frac{1}{2}\right)^k\cdot(k+1)\,u[k]$

11.13 i) $$y[k] = \begin{cases} \frac{4^{k+1}}{3}, & k < 0 \\ \frac{5}{3} - \frac{1}{3}\left(\frac{1}{4}\right)^k, & k \geq 0 \end{cases}$$

ii) $$y[k] = \frac{6}{5} 3^k u[4-k] + \frac{3^5}{5} \cdot 2^{-(k-5)} u[k-5]$$

11.19 i) $$H(\Omega) = -\frac{\left(1+\frac{1}{4}e^{-j\Omega}\right)\left(1-\frac{1}{3}e^{-j\Omega}\right)}{\left(1-\frac{7}{24}e^{-j\Omega}\right)\left(1-\frac{3}{4}e^{-j\Omega}\right)}$$

ii) $$h[k] = \frac{8}{21}\delta[k] - \frac{13}{77}\left(\frac{7}{24}\right)^k u[k] - \frac{40}{33}\left(\frac{3}{4}\right)^k u[k]$$

iii) $$y[k] - \frac{25}{24}y[k-1] + \frac{7}{32}y[k-2] = -x[k] + \frac{1}{12}x[k-1] + \frac{1}{12}x[k-2]$$

iv) Ja, eftersom $h[k] = 0 \ \forall \ k < 0$.