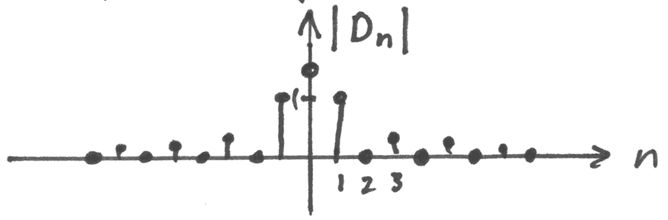
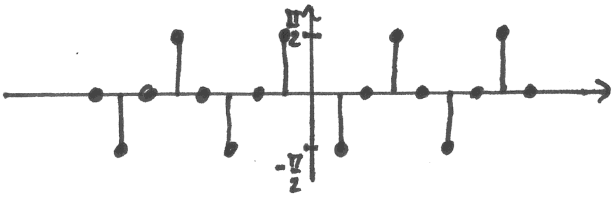
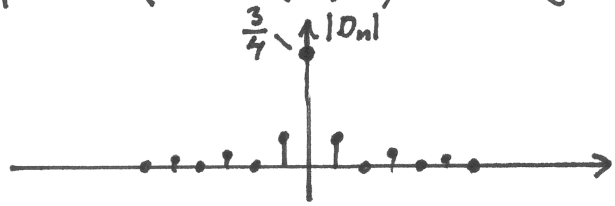
TSKS21 Facit till lektion 5

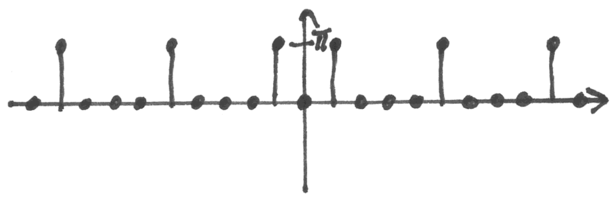
4.11 a)
$$D_n = \frac{3}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right)\cdot e^{-jn\pi/2} = \begin{cases} \frac{3}{2}, & n=0 \\ \frac{3}{n\pi}\, e^{-jn\pi/2}\cdot \sin\left(n\frac{\pi}{2}\right), & n\neq 0 \end{cases}$$





b)
$$D_n = \frac{1}{2}\left(\frac{1}{2} + (-1)^n\right)\mathrm{sinc}\left(\frac{n}{2}\right)$$





5.1 c)
$$X_3(\omega) = \frac{a+j\omega}{(a+j\omega)^2 + \omega_0^2}$$

5.2 a)
$$X_1(\omega) = 3\pi\cdot \mathrm{sinc}\left(\frac{\omega}{2}\right) e^{-j\pi\omega/2} \quad \text{(jfr. 4.11a)}$$

b)
$$X_2(\omega) = T\cdot\left(\frac{1}{2} + e^{-j\omega T}\right)\cdot \mathrm{sinc}\left(\frac{\omega T}{2\pi}\right) \quad \text{(jfr. 4.11b)}$$

e)
$$X_5(\omega) = \left(T\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right) - \frac{\pi}{T}\cdot\frac{\cos(\omega T/2)}{(\pi/T)^2 - \omega^2}\right)\cdot e^{-j\omega\frac{T}{2}},$$

där gränsvärdet gäller i $\omega = \pm\pi/T$.

5.4 a)
$$x_1(t) = \left(2e^{-3t} - 3e^{-2t}\right)\cdot u(t)$$

b)
$$x_2(t) = \left(\frac{3}{2}e^{-t} - 4e^{-2t} + \frac{5}{2}e^{-3t}\right)\cdot u(t)$$

d)
$$x_4(t) = (1-\cos(t))\cdot e^{-t}\cdot u(t)$$

5.9 a) $X_1(\omega) = 10\pi\delta(\omega) + 3\pi\big(\delta(\omega-10) + \delta(\omega+10)\big) - \dfrac{21}{(2+j\omega)^2 + 9}$

b) $X_2(\omega) = -j\cdot \mathrm{sgn}(\omega)$

c) $X_3(\omega) = \left(-\dfrac{200}{16+\omega^2} + \dfrac{j160\omega - 16}{(16+\omega^2)^2} + \dfrac{64\omega^2}{(16+\omega^2)^3}\right)\cdot e^{-j5\omega}$

5.10 —

5.20 a) $H(\omega) = \dfrac{1}{(j\omega)^3 + 6(j\omega)^2 + 11j\omega + 6} = \dfrac{1}{(j\omega+1)(j\omega+2)(j\omega+3)}$

$h(t) = \left(\frac{3}{2}e^{-t} - 4e^{-2t} + \frac{5}{2}e^{-3t}\right)\cdot u(t)$ (samma som 5.4b)

b) $H(\omega) = \dfrac{1}{(j\omega)^2 + 3j\omega + 2} = \dfrac{1}{(j\omega+1)(j\omega+2)}$

$h(t) = (e^{-t} - e^{-2t})\cdot u(t)$

c) $H(\omega) = \dfrac{1}{(j\omega)^2 + 2j\omega + 1} = \dfrac{1}{(j\omega+1)^2}$

$h(t) = te^{-t}u(t)$

d) $H(\omega) = \dfrac{j\omega+4}{(j\omega)^2 + 6j\omega + 8} = \dfrac{j\omega+4}{(j\omega+2)(j\omega+4)} = \dfrac{1}{j\omega+2}$

$h(t) = e^{-2t}u(t)$

5.21 a) $H(\omega) = 5$, $h(t) = 5\delta(t)$, $y(t) = 5x(t)$

b) $H(\omega) = 3e^{-j4\omega}$, $h(t) = 3\delta(t-4)$, $y(t) = 3x(t-4)$

c) $H(\omega) = \dfrac{6}{(2+j\omega)^3}$, $h(t) = 3t^2e^{-2t}u(t)$

$\left(\dfrac{d^3}{dt^3} + 6\dfrac{d^2}{dt^2} + 12\dfrac{d}{dt} + 8\right)y(t) = 6x(t)$

5.21 d) $H(\omega) = \frac{2(j\omega+2)^2}{(j\omega+1)(j\omega+3)} = 2 + \frac{2}{(j\omega+1)(j\omega+3)}$

$$h(t) = 2\delta(t) + (e^{-t} - e^{-3t})u(t)$$

$$\left(\frac{d^2}{dt^2} + 4\frac{d}{dt} + 3\right)y(t) = \left(2\frac{d^2}{dt^2} + 8\frac{d}{dt} + 8\right)x(t)$$

5.22 $H(\omega) = \frac{1}{1+j\omega RC}$

$$y(t) = \begin{cases} 0, & t < -T/2 \\ 1 - e^{-(t+\frac{T}{2})/RC}, & |t| \leq T/2 \\ e^{-(t-\frac{T}{2})/RC} - e^{-(t+\frac{T}{2})/RC}, & t > T/2 \end{cases}$$