TSKS04 Digital Communication
Continuation Course

Lecture 9

Linear Equalization, Block codes, Convolutional codes

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Linear Equalization

Time-discrete model

$$\boldsymbol{x}[n] = \boldsymbol{U} \cdot \boldsymbol{B}[n] + \boldsymbol{w}[n]$$

Received signal vector $L \times 1$

Matrix $L \times K$

Symbol vector $K \times 1$

$$\boldsymbol{B}[n] = (b[n-k_1], \dots, b[n], \dots, b[n+k_2])^T$$

$$K = k_1 + k_2 + 1$$

Noise vector $L \times 1$

Linear equalization

- Take inner product between $\boldsymbol{x}[n]$ and some vector $\boldsymbol{c}$
- Select the $L \times 1$ vector $\boldsymbol{c}$ so that $\boldsymbol{c}^H \boldsymbol{x}[n]$ gives good estimate of $b[n]$

Different Linear Equalization Choices (1/2)

- Matched filter (MF) equalization
  - Maximize desired signal: $\boldsymbol{c}^H \boldsymbol{u}_0$
  - Fix $\boldsymbol{c}^H \boldsymbol{u}_0 = 1$: $\boldsymbol{c} = \boldsymbol{u}_0 / \|\boldsymbol{u}_0\|^2$

- Zero-forcing (ZF) equalization
  - Force ISI to be zero: $\boldsymbol{c}^H \boldsymbol{U} = (\underbrace{0 \; \ldots \; 0}_{k_1} \; 1 \; \underbrace{0 \; \ldots \; 0}_{k_2}) = \boldsymbol{e}^T$
  - Achieved by $\boldsymbol{c}_{ZF} = \boldsymbol{U}(\boldsymbol{U}^H \boldsymbol{U})^{-1} \boldsymbol{e}$

Different Linear Equalization Choices (2/2)

- Comparison
  - MF: Maximize desired signal
  - ZF: Remove inter-symbol interference
  - What one to choose?

- Minimum mean squared error (MMSE) equalization
  - Balance between strong signal and little interference
  - Mean squared error:

$$J(\boldsymbol{c}) = E\left[\left|\boldsymbol{c}^H \boldsymbol{x}[n] - b[n]\right|^2\right]$$

$$\boldsymbol{c}_{MMSE} = \sigma_b^2 \left(\sigma_b^2 \boldsymbol{U}\boldsymbol{U}^H + \boldsymbol{C}_w\right)^{-1} \boldsymbol{u}_0$$

Noise covariance matrix

Outlook: Equalization of Multi-user Channels

$$\boldsymbol{x}[n] = \boldsymbol{U} \cdot \boldsymbol{B}[n] + \boldsymbol{w}[n]$$

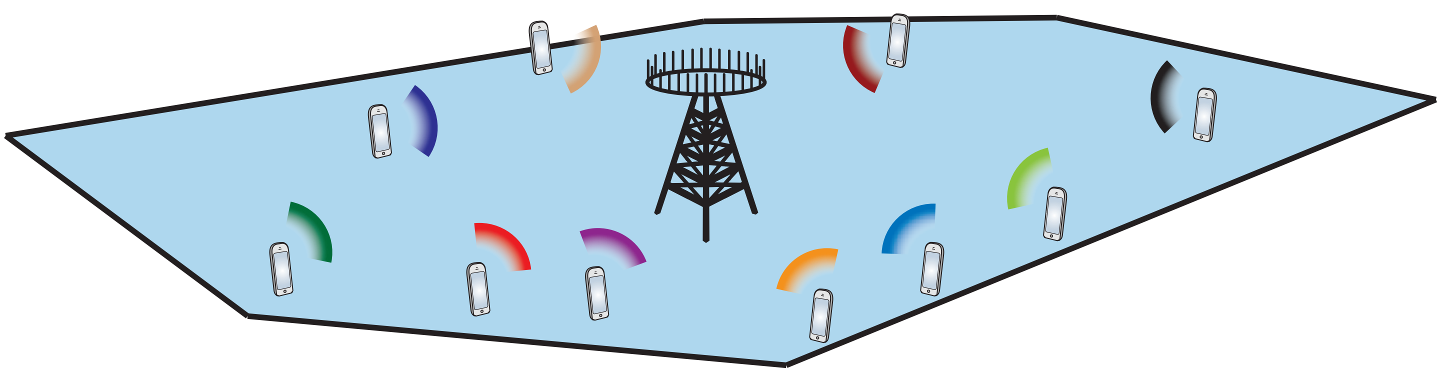
Received signal vector $L \times 1$

Matrix $L \times K$

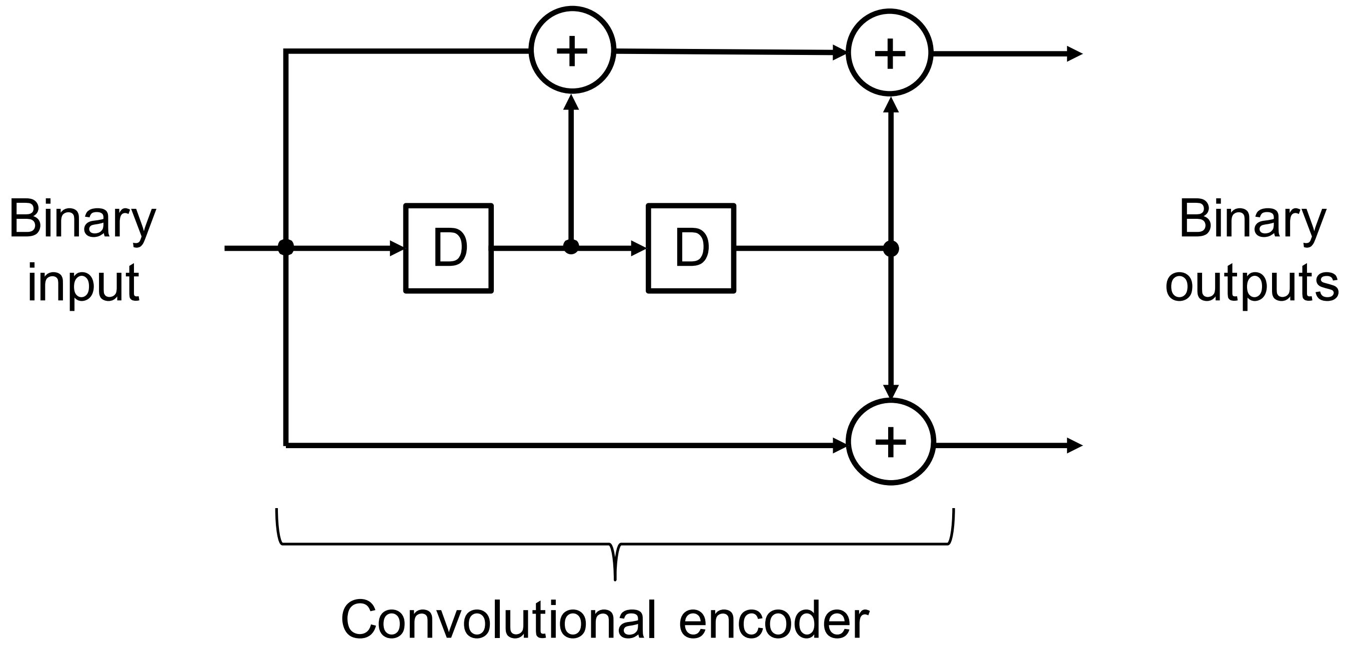
Symbol vector $K \times 1$

Noise vector $L \times 1$

- Two scenarios, same equations:
  - Single-user channel with ISI: K symbols and L samples
  - Multi-user channel without ISI: K users and L receive antennas

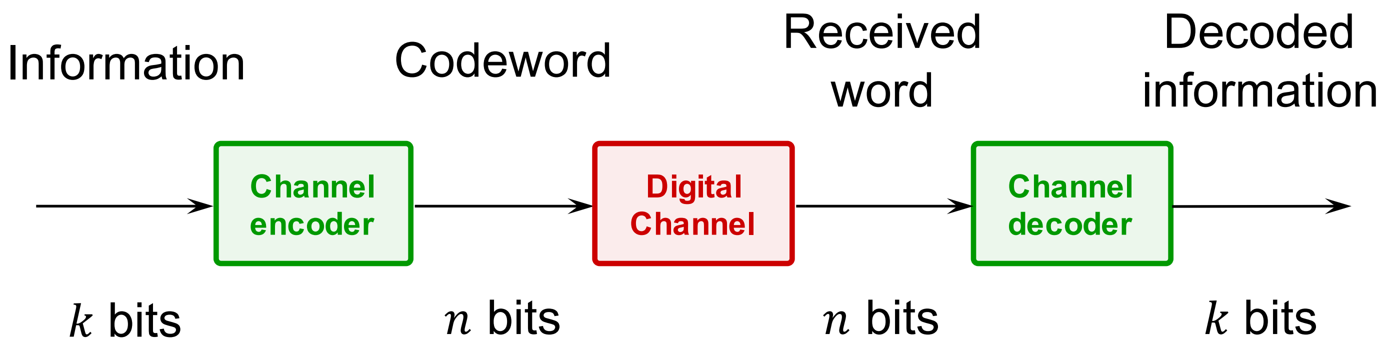


Introduction: Convolutional Codes

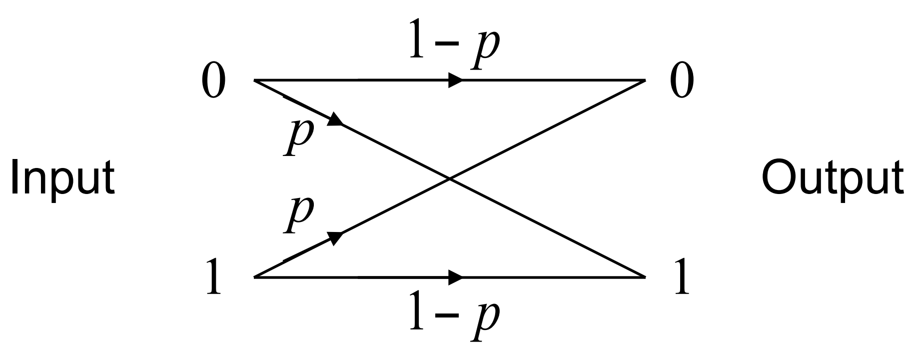


- Encoder: FIR filter, convolution
- State diagram, trellis
- Decoding: Viterbi algorithm

Repetition: Error Control Coding



- Simple channel model: Bits in, bits out
  - Example: Binary symmetric channel



$p =$ Bit error probability

Repetition: Binary Linear Codes $[n, k, d]$

A vector space expressed in a basis

$$\mathcal{C} = \{\bar{m}G \ \ \forall \bar{m} \in \mathbb{F}_2^k\}$$

Generator matrix $(k \times n)$, linearly independent rows.

… the nullspace of a matrix

$$\mathcal{C} = \{\bar{c} \in \mathbb{F}_2^n : H\bar{c}^T = \bar{0}\}$$

Parity check matrix $((n - k) \times n)$, linearly independent rows.

$$HG^{\mathrm{T}} = 0$$

Length: n, # columns in G or H

Dimension: k, # rows in G.

<u>Minimum distance, d</u>

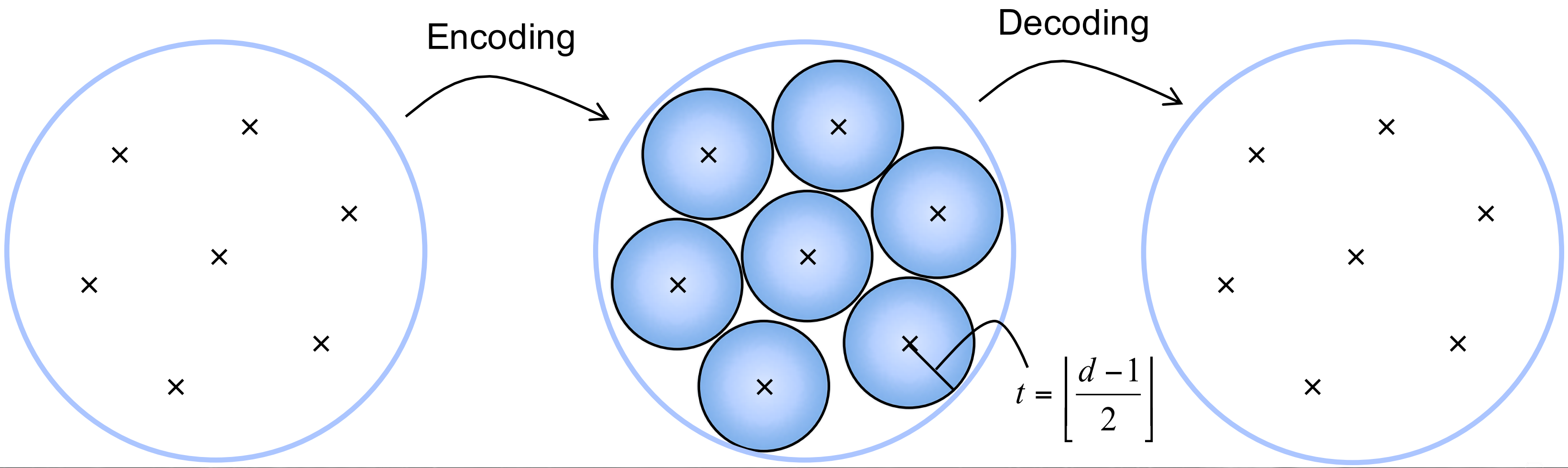
Smallest Hamming distance between different codewords.

Smallest Hamming weight of non-zero codewords.

Smallest number of linearly dependent columns in H.

Repetition: Decoding of block codes

- Syndrome decoding
  - Parity check matrix: $H\bar{c}^{\mathrm{T}} = \bar{0}$
  - Syndrome: $\bar{s} = H\bar{x}^{\mathrm{T}} = H \cdot (\bar{c} + \bar{e})^{\mathrm{T}} = \underbrace{H\bar{c}^{\mathrm{T}}}_{=\bar{0}} + H\bar{e}^{\mathrm{T}} = H\bar{e}^{\mathrm{T}}$
  - **Syndrome decoding**: Find smallest set of columns in H that sum up to the syndrome



$$t = \left\lfloor \frac{d-1}{2} \right\rfloor$$

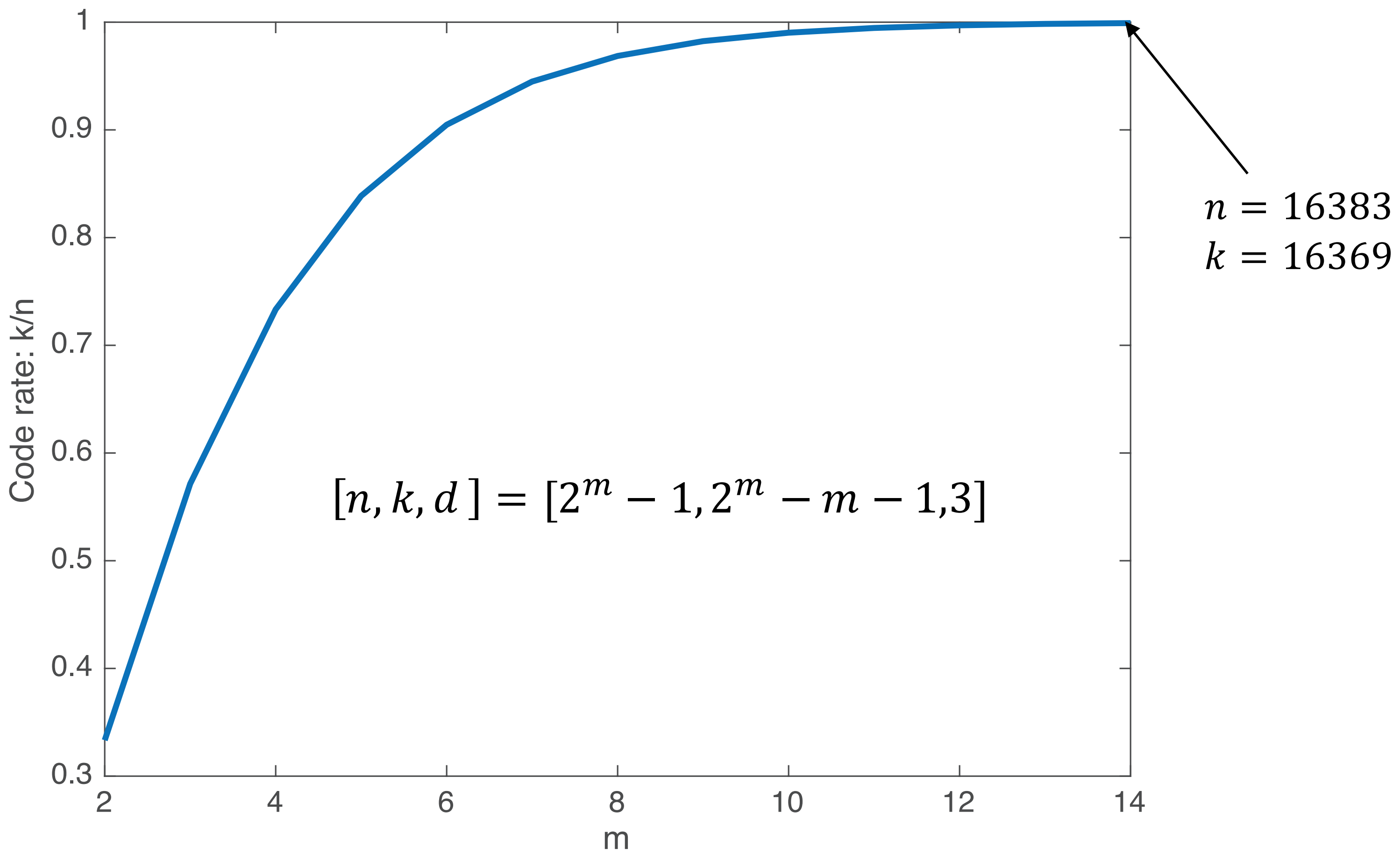
What is a Good Block Code? (1/2)

Consider: A *Hamming code*

- Defined by $m \geq 2$.
- Parity check matrix contains all non-zero binary vectors of length m.
- Gives: $n = 2^m - 1, \qquad k = 2^m - m - 1$

$$[n, k, d] = [2^m - 1, 2^m - m - 1, 3]$$

What is a Good Block Code? (2/2)



Long block codes: Can achieve a given d with almost code rate 1

Binary Sequences

Consider: $a = a_0, a_1, a_2, \ldots$ for $a_i \in \mathbb{F}_2$

- D-transform: Write sequence using delay operator D
- $A(D) = a_0 + a_1 D + a_2 D^2 + \cdots$

Convolution of binary sequences:

- $P(D) = p_0 + p_1 D + p_2 D^2 + \cdots + p_m D^m$
- Consider: $X(D) = A(D)P(D)$
- Time representation $x = a * p$:

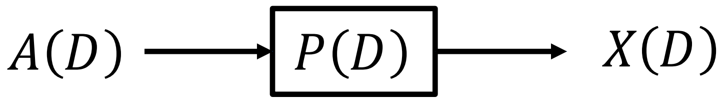
$$x_m = \sum_{k=0}^{m} a_k p_{m-k} = \sum_{k=0}^{m} a_{m-k} p_k$$

Filtering of Sequences

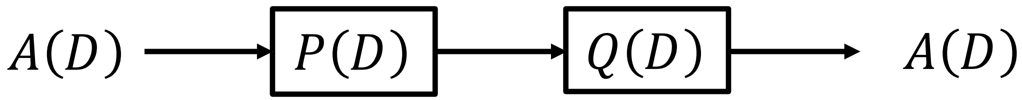
Input sequence: $A(D) = a_0 + a_1 D + a_2 D^2 + \cdots$

Filter p:

- Impulse response: $p = p_0, \ldots p_m$
- Transfer function: $P(D) = p_0 + p_1 D + p_2 D^2 + \cdots + p_m D^m$



Inverse filter q:



- Delay-less inverse q exist if and only if $p_0 = 1$
- Delayed inverses are also possible

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