TSKS04 Digital Communication

Continuation Course

Lecture 6

Non-coherent communication, Channel Equalization

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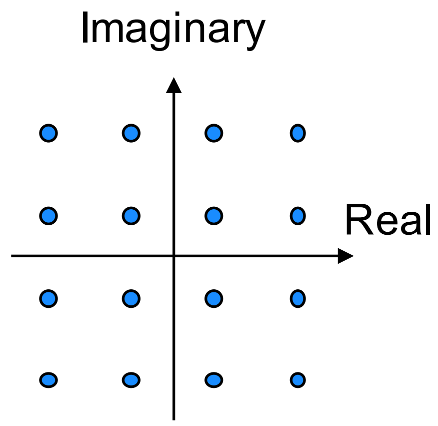
Coherent Communication

- Recall: Model for M-ary coherent communication
  - Send one of $s_1(t), \dots, s_M(t)$ (basis function × constellation point)
- Complex-baseband interpretation
  - Two-dimensional constellation
  - Use real and imaginary part



- Detection problem solved in TSKS01:
  - Receive a noisy signal $y(t)$
  - $H_i: \quad y(t) = s_i(t) + n(t)$
  - Detect the signal $s_i(t)$

ML detection:
Matched filtering + closest constellation point

Non-Coherent Communication

- Model for M-ary non-coherent communication
  - Send one of $s_1(t), \ldots, s_M(t)$ (basis function × constellation point)
  - The phase θ of the signal is unknown
  - Detect signal $s_i(t)$ from an observation $y(t)$

Hypotheses:

$$H_i: \quad y(t) = s_i(t)e^{j\theta} + n(t)$$

How to estimate the signal
when the phase is unknown?

Composite Hypothesis Testing (1/2)

Consider M hypotheses regarding observation y

- Denoted H_i for $i = 1, \dots, M$
- Conditional distribution $p(y|i,\theta)$ depends on unknown θ

Composite hypothesis test:

$$(\hat{\imath}, \hat{\theta})(y) = \underset{1 \leq i \leq M, \theta}{\operatorname{argmax}} \; p(y|i,\theta)$$

- Step 1: For each i, find $\hat{\theta}_i(y) = \operatorname{argmax}_\theta \; p(y|i,\theta)$
- Step 2: Define $q(y|i) = p(y|i, \hat{\theta}_i(y))$ ← **Not a true density** (integral $\neq 1$)
- Step 3: Solve a generalized likelihood ratio test:

$$\delta_{GLRT}(y) = \operatorname{argmax}_{1 \leq i \leq M} \; q(y|i)$$

Composite Hypothesis Testing (2/2)

Hypothesis testing with prior information

- H_i: The signal is $s_i(t)$ occurs with probability $\pi(i)$
- Likelihood function $L(y|i,\theta)$ depends on unknown θ
- Bayes' rule:

$$p(i|y,\theta) = \frac{L(y|i,\theta)\pi(i)}{p(y)}$$

Composite hypothesis test:

- Step 1: For each i, find $\hat{\theta}_i(y) = \mathrm{argmax}_\theta \; p(i|y,\theta)$
- Step 2: Define $q(i|y) = p(i\,|y,\hat{\theta}_i(y))$ ← **Not a true density** (integral $\neq$ 1)
- Step 3: Solve a generalized likelihood ratio test:

$$\delta_{GLRT}(y) = \mathrm{argmax}_{1\le i\le M} \; q(i|y)$$

Non-Coherent Communication: Summary

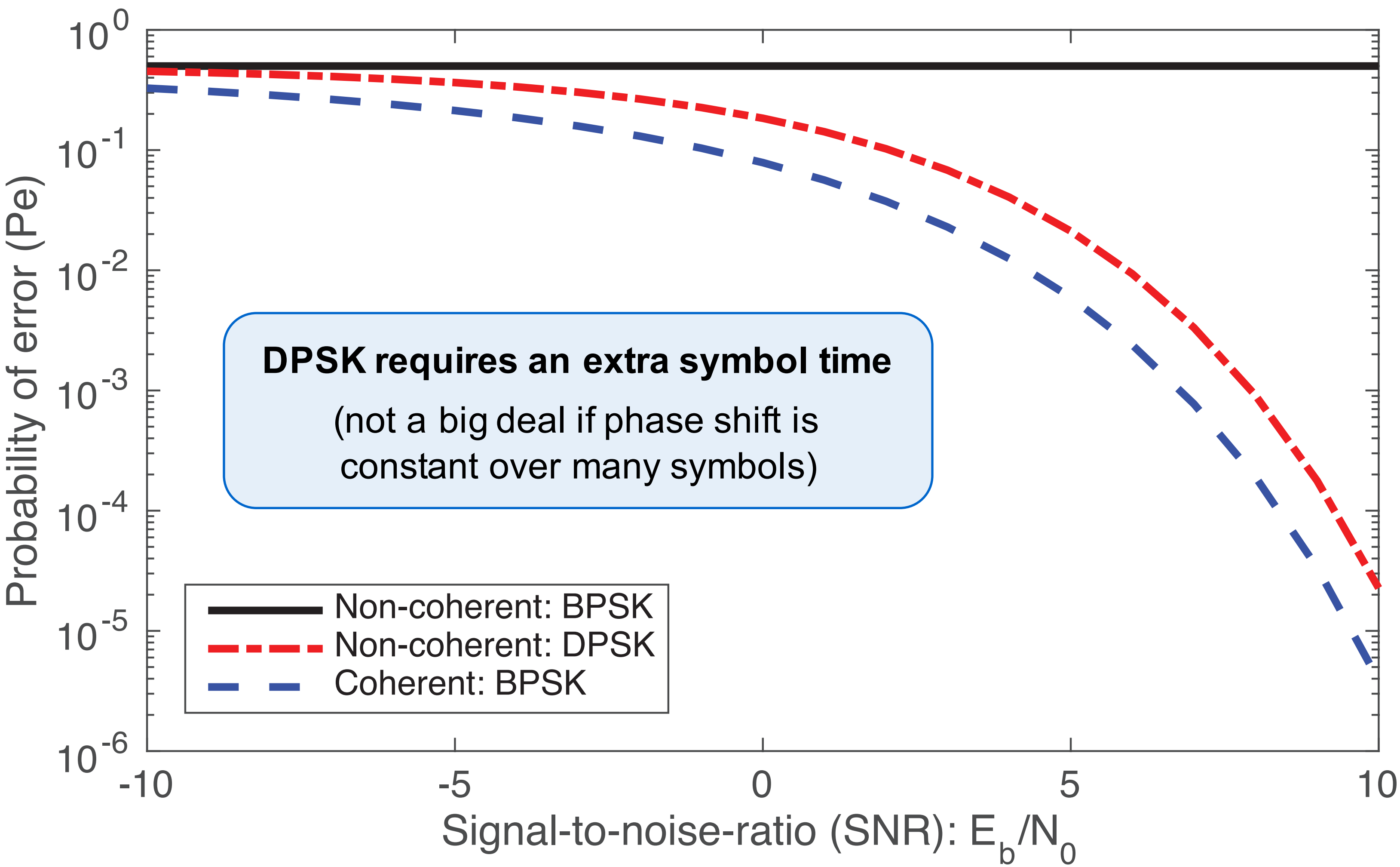
ML detection rule:

- Coherent: $\hat{\imath} = \mathrm{argmax}_{1 \le i \le M} \; Re\{z_i\} - \frac{E_i}{2}$
- Non-coherent: $\hat{\imath} = \mathrm{argmax}_{1 \le i \le M} \; |z_i| - \frac{E_i}{2}$

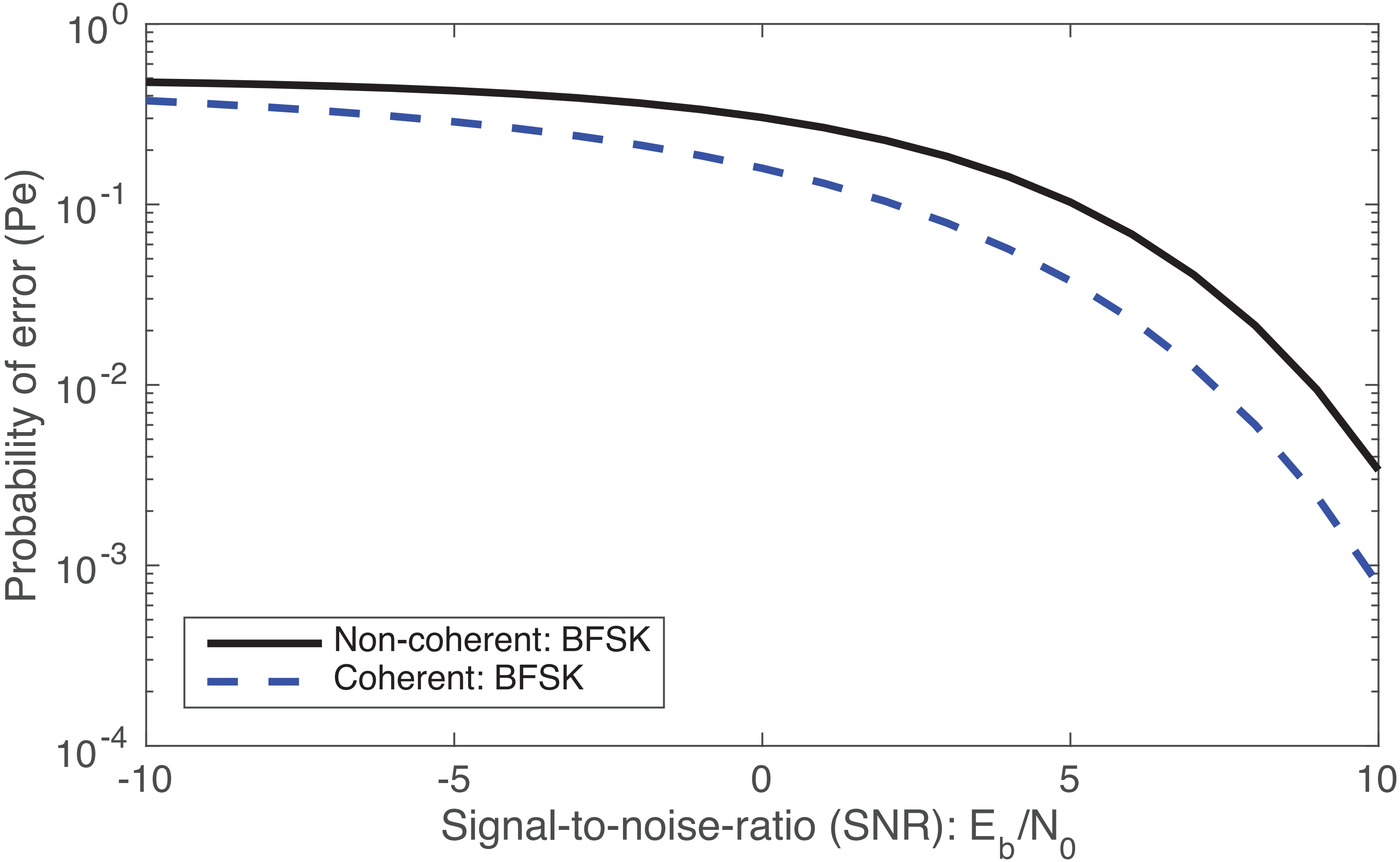
$z_i = \langle y, s_i \rangle$ $\quad\quad$ $E_i = \langle s_i, s_i \rangle = \|s_i\|^2$

- Consequences of non-coherence
  - Cannot separate constellation points that have different phase shift
  - Can still separate signals along orthogonal basis functions
  - More noise: Consider both real and imaginary part

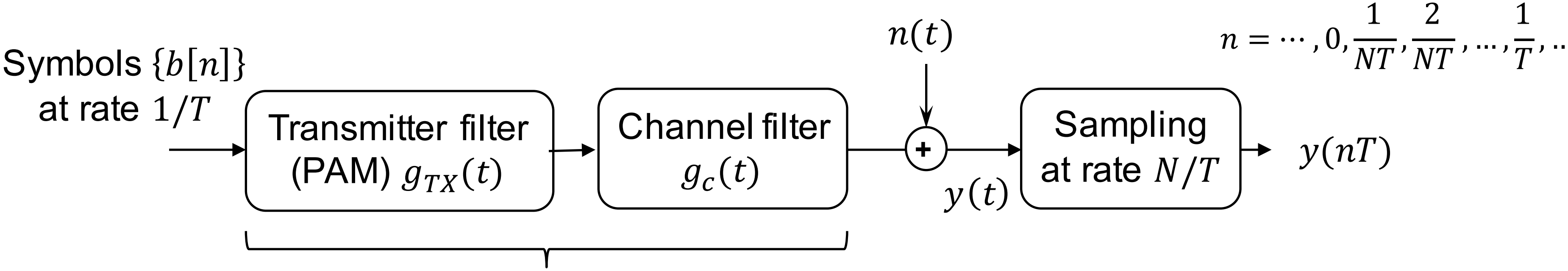
Performance: Coherent vs. Non-Coherent



Performance: Coherent vs. Non-Coherent



Channel Equalization (1/4)



Effective pulse shape:
$p(t) = (g_{TX} * g_c)(t)$

Received analog signal:

$$y(t) = \sum_m b[m] p(t - mT) + n(t)$$

N-times over-sampling of received signal

- Good resolution for digital filtering
- No need to implement a “good” analog filter

Suppose we know $y(t)$: What is the best receiver filter?

Channel Equalization (2/4)

Hypothesis: Sequence $\{b[n]\}$

- Step 1: Filter $y(t)$ with $h_{RX}(t) = p^*(-t)$ (a matched filter)
- Step 2: Sample $(y * h_{RX})(t)$ at $t = nT$ to get $z[n]$
- Step 3: Correlate with hypothesis sequence $\{b[n]\}$

Learn $p(t)$: Use channel estimation framework

ML criterion:

$$\hat{\boldsymbol{b}} = \text{argmax}_{\boldsymbol{b}} \ \ln L(y|\boldsymbol{b})$$

where

$$\ln L(y|\boldsymbol{b}) = \frac{1}{\sigma^2}\left(\sum_m Re\{z[m]b^*[m]\} - \frac{\|s_{\boldsymbol{b}}\|^2}{2}\right)$$

Channel Equalization (3/4)

Terminology: *Channel equalization*

- Reverse distortion caused by channel
- In particular: Inter-symbol interference and noise

Have solved the equalization problem?

- Yes, we can try all $\{b[n]\}$ and see which one that maximizes $L(y|\boldsymbol{b})$

Example: M-ary constellation and sequence $b[0], \ldots, b[L-1]$

- M^L different sequences
- $L = 10$ and $M = 16$: $16^{10} \approx 1.1$ trillion sequences

Unreasonable complexity (and delay)!

Channel Equalization (4/4)

ML criterion:

$$\ln L(y|\boldsymbol{b}) = \frac{1}{\sigma^2}\left(\sum_m Re\{< y, s_b >\} - \frac{\|s_{\boldsymbol{b}}\|^2}{2}\right)$$

Can we update ML rule symbol by symbol?

- First term: $Re\{< y, s_b >\} = \sum_m Re\{z[m]b^*[m]\}$ Yes!
- Second term:

$$\|s_{\boldsymbol{b}}\|^2 = \sum_m \left(|b[m]|^2 h[0] + \sum_{n:n<m} 2Re\{b[n]b^*[m]h(m-n)\}\right)$$

where $h(n) = r_p(Tn)$ and $r_p(\tau) = \int_{-\infty}^{\infty} p(t)p^*(t-\tau)dt$ Yes!

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