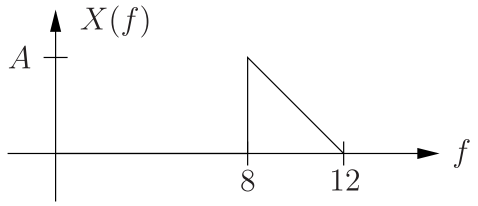
Tutorial 3

1 A real-valued bandpass signal $x(t)$ with spectrum $X(f)$ is plotted below for positive frequencies.



Notice that this spectrum is real-valued.

a. Sketch $X(f)$ for both positive and negative frequencies.

b. Sketch the spectrum $\tilde{X}(f)$ of the complex baseband represesentation of $x(t)$ with respect to the carrier frequency $f_c = 10$ Hz.

c. Is there any carrier frequency such that the complex baseband represesentation of $x(t)$ is real?

2 An ideal BP filter with frequency response

$$H(f) = \begin{cases} 1, & 3 < |f| < 5, \\ 0, & \text{elsewhere}, \end{cases}$$

is to be implemented using demodulation, baseband filtering and modulation. Determine the frequency response of the baseband filter for the following two cases.

a. Carrier frequency $f_c = 4$ Hz.

b. Carrier frequency $f_c = 5$ Hz.

3 Consider a complex baseband signal $\tilde{x}(t)$ and a complex baseband filter with impulse response $\tilde{h}(t)$. Implement this filter as a block diagram using only real-valued baseband operations, i.e. filters, amplifiers and adders.

4 Consider a bandpass signal $x(t)$ with passband $2 < |f| < 6$. In a communication equipment, we want to move that spectrum to the new passband $4 < |f| < 8$. This can be achieved by demodulation and modulation via a baseband representation of $x(t)$. Specify all details of this operation and give a block diagram.

5 The Fourier transform $\tilde{X}(f)$ of the baseband representation $\tilde{x}(t)$ of a passband signal $x(t)$ with Fourier transform $X(f)$ is given by

$$\tilde{X}(f) \quad = \quad (2\pi f + j)\, I_{\{|f|<1\}}(f).$$

For the carrier frequency, we have $f_\mathrm{c} > 1$. Determine $\tilde{x}(t)$, $x(t)$ and $X(f)$.

6 This task is about designing what we could call a spectrum inverter. We want an equipment that takes as input a bandpass signal $x(t)$ with Fourier transform $X(f)$, and produces as output a bandpass signal $y(t)$ with Fourier transform $Y(f)$ according to

$$Y(f) = \begin{cases} X(f - 2f_\mathrm{c}), & f_\mathrm{c} - B < f < f_\mathrm{c} + B, \\ X(f + 2f_\mathrm{c}), & -f_\mathrm{c} - B < f < -f_\mathrm{c} + B, \\ 0, & \text{elsewhere.} \end{cases}$$

The input is assumed to be strictly bandlimited to the frequency band $f_\mathrm{c} - B < |f| < f_\mathrm{c} + B$, where we have $B < f_\mathrm{c}$.

Design the equipment using modulations and filters. Any filtering has to be done in the baseband region. For simplicity, use ideal filters where needed.

Give a block diagram of your design using only real-valued signals and operations.