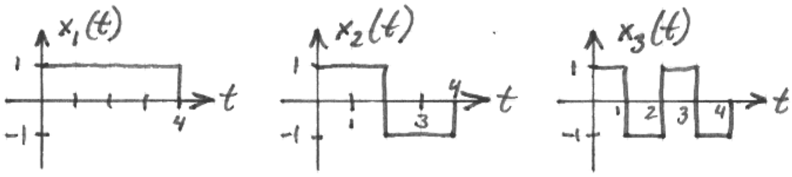
TSKS01 Digital Communication

Extra problems for Problem Class 1

Mikael Olofsson, mikael@isy.liu.se

1) Let $x_1(t)$, $x_2(t)$, and $x_3(t)$ be the signals given below.



a) Determine impulse responses $h_1(t)$, $h_2(t)$ and $h_3(t)$ for filters that are matched to those signals.

b) Determine the outputs of those filters when the input is the signal that the filter is matche to, i.e determine

$$(x_1 * h_1)(t), \quad (x_2 * h_2)(t), \quad (x_3 * h_3)(t)$$

Especially, note the value of the ouputs at $t = 4s$.

c) Determine the following

$$(x_1 * h_2)(t), \quad (x_2 * h_3)(t), \quad (x_3 * h_1)(t)$$

Especially, note the value of the outputs at $t = 4s$.

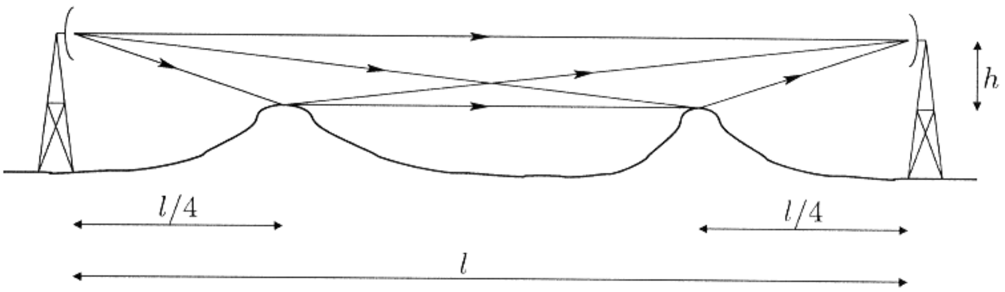
2) Consider a five meters long cable with a characteristic impedance that is constant and real-valued, and assume that this cable is correctly terminated. The serial impedance of the cable is $r + j\omega l$ ohms/meter and the parallel admittance of the cable is $g + j\omega c$ siemens/meter.

Detererminе the power spectral density of the output, expressed in that of the output.

4 Telecommunication Channels – Problems

4.1 A coaxial cable has amplitude characteristic $|H(f)| = e^{-\sqrt{|f|/f_0}}$, where f_0 is determined by the dimensions of the cable. Determine the 3 dB cut-off frequency of this cable.

4.2 A radio link is used over somewhat rough terrain. The two peaks in the figure below dominate the reflections. We therefore model our radio link as if those reflections were the only ones. The reflection coefficient of both reflection points is ρ. Determine the transfer function of the channel taking all four paths into account. For simplicity, disregard the attenuation of the signal due to the transmission through space.



4.3 Consider a step-index fiber, for which the core has refraction index 1.5, the cladding has refraction index 1.4, and the surrounding media is air.

a. Light in the core reaches the junction between the core and the cladding with an angle of incidence that is 45°. Determine the angle of refraction.

b. Determine the critical angle of the core/cladding junction.

c. Determine the acceptance half-angle of the fiber.

d. Determine the numeric aperture of the fiber.

4.4 Consider a step-index fiber, for which the core has refraction index 1.5. The fiber is supposed to be used with air as the surrounding media, and the numeric aperture is 0.3 for that case.

a. Determine the refraction index of the cladding.

b. Determine the acceptance half-angle of the fiber.

c. Assume that the same fiber is used with water ($n = 4/3$) as surrounding media. Determine the acceptance angle and the numeric aperture for that case.

4 Telecommunication Channels – Hints

4.1 The 3 dB cut-off frequency: $|H(f)| = H_{\max}/\sqrt{2}$, with $H_{\max} = \max |H(f)|$.

4.2 Pythagoras.

4.3 Snell's law.

4.4 Snell's law.

4 Telecommunication Channels – Answers

4.1 $0.12 f_0$

4.2 $H(f) = e^{-j2\pi fl/c} + 2\rho e^{-j2\pi fl_1/c} + \rho^2 e^{-j2\pi fl_2/c}$,
with $l_1 = \sqrt{\frac{9}{16}l^2 + h^2} + \sqrt{\frac{1}{16}l^2 + h^2}$
and $l_2 = \frac{l}{2} + 2\sqrt{\frac{1}{16}l^2 + h^2}$.

4.3 **a.** 49.3° **b.** 69.0° **c.** 32.6° **d.** 0.539

4.4

a. 1.47

b. 17.5°

c. Acceptance half-angle: 13.0°
Numeric aperture: 0.225